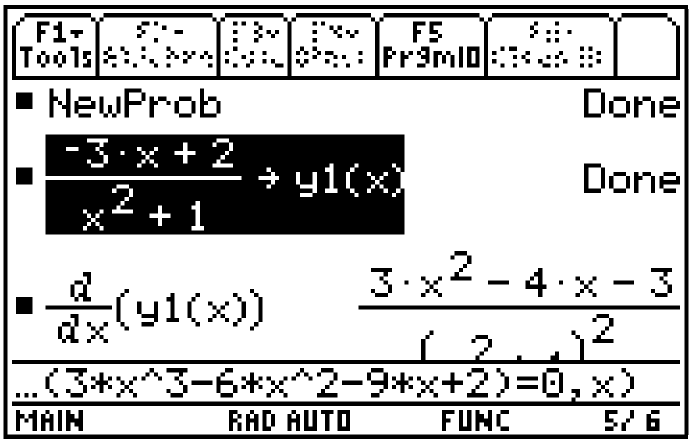
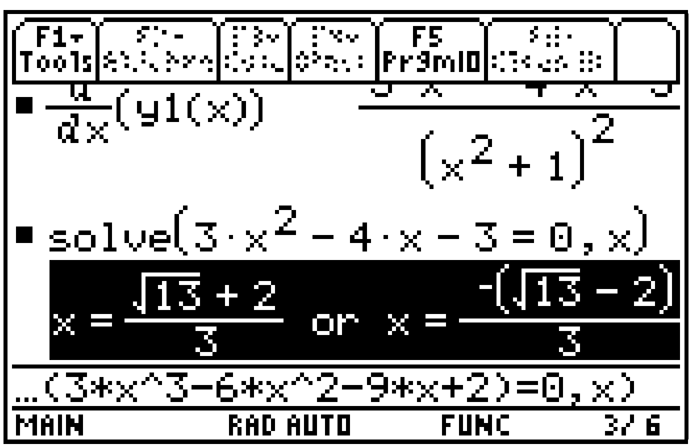
Using the TI89 Integlligently to Help with Graphing

In this example we use the TI89 to help analyze the graph of the function f(x)=(-3x+2)/(x^2+1). We will not do all the analysis here but will show screen shots from the TI89 so you can see how to get the calculator to do some of the grunge work while at the same time getting the kind of information you need to give a good argument as to why you state that there is a local extremum or an inflection point at a certain place in the graph.

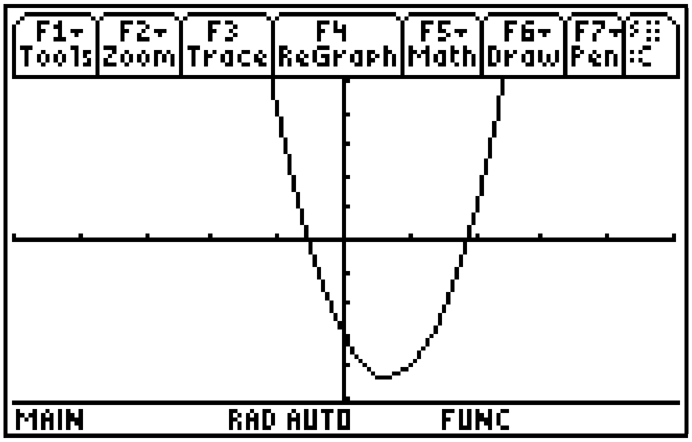
First we tell the machine what the function is.



Next we ask the machine for the derivative of the function and we notice that the sign of the first derivative is determined entirely by a the quadratic polynomial in the numerator of the first derivative. (That will not always be the case! This is where you have to use your head.) We ask the machine for the critical numbers for the function by having it solve the equation you get by setting the numerator of the first derivative equal to zero.

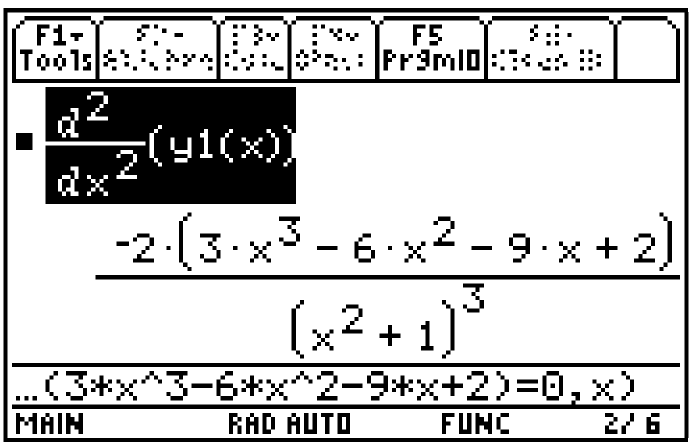


By grahping the numerator of f', we can see more precisely where f'is positive and negative, thus where f is respectively increasing and decreasing.



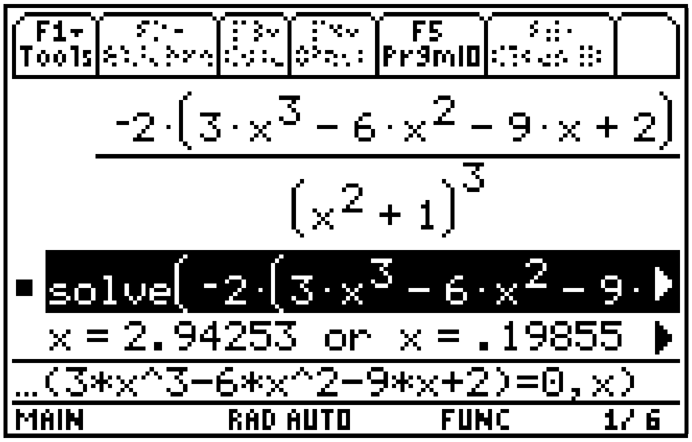
Now we can say that f is increasing on the intervals from minus infinity to –(sqrt{3}-2)/3 and (sqrt{3}+2)/3 to infinity. (Those are the intervals where f'is positive.) We can say that f is decreasing on the interval from –(sqrt{3}-2)/3 to (sqrt{3}+2)/3, that is, where f'is negative. We can also conclude that f has a local maximum where x= –(sqrt{3}-2)/3 and that f has a local minimum where x=(sqrt{3}+2)/3.

The second derivative of f tells us about the concavity of the graph of f. We ask the machine to calculate the second derivative.

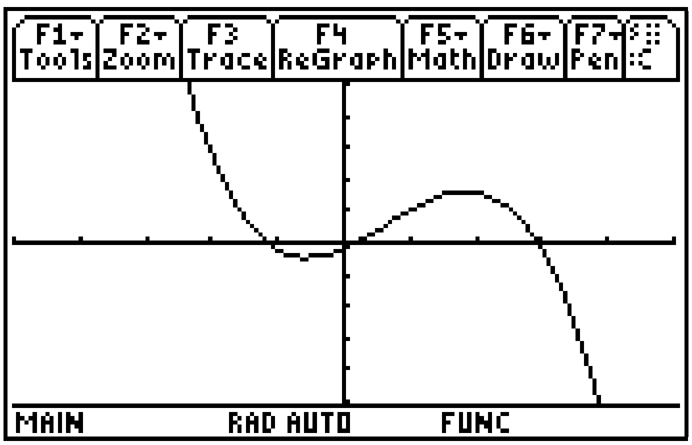


Note, again, that the sign of the second derivative is determined by the numerator because the denominator is always positive. This will not always be the case. You have to make observations and judgments when you do these problems.

We ask the calculator to solve the equation we get by setting the numerator of f'' equal to zero. This gives us possible inflection points, that is, points where the graph changes concavity.



As the equation is a cubic, we are not surprised to get three points where f’’(x)=0. Again, we can graph the numerator of f’’ to see exactly where it is positive and where it is negative, thus determining where f is respectively concave up and concave down. The graph of the cubic looks like this.



Now we know that all three solutions to the previous equation do indeed correspond to inflection points on the graph of f because f changes from concave up, to concave down, to concave up, and finally to concave down, just as f’’ changes from positive, to negative, to positive, to negative.