

Math 2254-05 Test #1 February 19, 1999

Directions

You must show your work on this test paper. Do not use scrap paper. When you use your calculator, indicate how you use it, e.g., "I used my calculator to find $y' = 2^x$."

1. (5 pts) Find $\frac{d}{dx}(\sin^{-1}(x^2y))$. An answer alone is sufficient.
2. (7 pts) Let $\frac{dQ}{dt} = 300 - 0.3Q$. Solve the differential equation subject to $Q(0) = 500$.
3. (8 pts) Let $f(x) = \frac{3x - 7}{5x + 11}$. **Use calculus** to argue that f is invertible.
4. (7 pts) Find the inverse of $f(x) = (1.05)^x$.
5. (8 pts) Let $f(x) = \sqrt{12x + 13}$. Find $g'(23)$ if $g = f^{-1}$.
6. (10 pts) Sketch the function which is the inverse of $f(x) = 3/(2x + 5)$. Label intercepts and give equations for asymptotes.

7. (20 pts) Match the following differential equations with their solutions.
Note: the given functions may satisfy more than one equation or none and some equations may have more than one solution or no solution.

(a) $y'' = y$	(I) $y = \cos x$
(b) $y' = -y$	(II) $y = \sin(2x)$
(c) $y' = 1/y$	(III) $y = x^2$
(d) $y'' = -y$	(IV) $y = e^x + e^{-x}$
(e) $y'' - 4y = 0$	(V) $y = \sqrt{2x}$

8. An exponentially decaying substance was weighed every hour and the results are given below.

Time	Weight (in grams)
9 am	10.000
10 am	8.958
11 am	8.025
12 noon	7.189
1 pm	6.440

- (a) (7 pts) Determine a formula of the form $Q = Q_0e^{-kt}$ which would give the weight of the substance, Q , at time t , where t is measured in hours since 9 am. Use your calculator, but indicate how you get to your answers. Give your numbers to three decimal places.
- (b) (8 pts) Find the half-life of this substance, accurate to three decimal places.
9. The population of a species of elk on a remote island has been monitored for some time. When the population was 600, the relative birth rate was 35% and the relative death rate was 15%. When the population was 800, the relative birth rate was 30% and the relative death rate was 20%. Assume there are no hunters and that there is no migration. Also assume that the relative growth rate (r.g.r.) is a linear function of the size of the population.
- (a) (10 pts) Write a differential equation to model the population as a function of time. (**Hint:** Recall that r.g.r. = $k - aP$ and that the logistic model is $\frac{1}{P} \frac{dP}{dt} = k - aP$.)

(b) (5 pts) We say that the population P has reached the carrying capacity of the environment when the population can grow no further, i.e., when $\frac{dP}{dt} = 0$. This is the largest population the environment can support. Find the largest population of elk the environment can support on this remote island.

(c) (5 pts) Sketch a rough graph of the population of elk as a function of time. How does the number you found in part (b) above figure into your sketch?