

2. The following numbers are the left, right, trapezoidal, and midpoint approximations to $\int_0^1 f(x) dx$, where $f(x)$ is as shown. (Each uses the same number of subdivisions.)

- I) 0.36735
- II) 0.39896
- III) 0.36814
- IV) 0.33575

(a) (16 pts) Which is which? **Justify your answer.**

(b) (5 pts) Write $A < \int_0^1 f(x) dx < B$ where $B - A$ is as small as possible. **Justify your answer.**

3. (20 pts) Consider the region bounded by the curve $y = e^x$, the x -axis, the line $x = 0$, and the line $x = 1$. Find the volume of the solid whose base is the given region and whose cross-sections, perpendicular to the x -axis, are isosceles right triangles with bases lying in the region.

4. A cylindrical rod 4 meters long is positioned along the positive x -axis with its left end at the origin. The rod has a variable density given by $\delta(x) = x^2 + 1$ kilograms per meter.

(a) (5 pts) Find the total mass of the rod.

(b) (8 pts) Find the center of mass of the rod, \bar{x} .

5. (20 pts) Write an integral to express the circumference of a circle with radius r . (Hint: Use a function to describe a semi-circle. How can an integral get you the circumference of the semi-circle? How can you get from that to the circumference of the whole circle?) Evaluate the integral to verify that you get what you would expect.