

Math 2254-05 Test #3 April 2, 1999

Directions

You must show your work on this test paper. Do not use scrap paper.

1. Let $r = f(\theta) = 3\cos(2\theta)$.

(a) (5 pts) Sketch the polar graph of $f(\theta)$.

(b) (5 pts) Find the **exact** area enclosed by the graph.

2. Let $r = f(\theta) = 2 + 4\cos\theta$.

(a) (15 pts) Calculate the **exact** area inside the large loop but outside the small loop in the polar graph of $f(\theta)$.

(b) (20 pts) Find the **exact** values of θ which yield horizontal tangents to the curve. Use this information to find the (x, y) coordinates of the highest point on the small loop, accurate to five decimal places.

3. Consider the polar curve given by $r = \theta$ on the interval $\theta = 0$, to $\theta = 4\pi$.

(a) (5 pts) Sketch the curve.

(b) (10 pts) Find the values of θ where the curve has vertical tangents.
Give your answers to 4 decimal places.

(c) (5 pts) Indicate in your sketch approximately where each of those vertical tangents will occur.

4. The following parametric equations define a “trochoid,” the path of a point inside a circle of radius r as it rolls along the x -axis. If the radius of the circle is r and the point’s initial position is $(d, 0)$ in the xy -plane, then the equations describing the curve are

$$x = r\theta - d \sin(\theta)$$

$$y = r - d \cos(\theta)$$

(a) (5 pts) Sketch the path of the trochoid with $r = 1$ and $d = 1/2$.

- (b) (10 pts) Write an integral in terms of r and d which gives the length of one arch of the trochoid if $0 < d < r$.
- (c) (5 pts) Evaluate this integral for $r = 1$ and $d = 1/2$. Your answer should be accurate to five decimal places.
- (d) (10 pts) Find an expression in terms of d and r which gives the area under one arch of the trochoid.
- (e) (5 pts) Find the exact area under one arch of the trochoid having $r = 1$ and $d = 1/2$.