# Math 2254-05 Test #4 April 27, 1999

## Directions

You must show your work on this test paper. Do not use scrap paper.

1. Determine which of the following converge and which diverge. For those which converge, determine the limit of the sequence. Show your work.

   (a) (5 pts) $\left\{\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty}$

   (b) (5 pts) $\{\sin(n)\}_{n=1}^{\infty}$

   (c) (5 pts) $\{1+(-1/3)^n\}_{n=1}^{\infty}$

   (d) (5 pts) $\left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots\right\}$

2. Determine which of the following converge and which diverge. For those which converge, determine the exact limit of the series. Show your work.

   (a) (5 pts) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$

(b) (5 pts) $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \ldots$

(c) (5 pts) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots$

(d) (5 pts) $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \frac{4}{81} + \ldots$

(e) (5 pts) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \ldots$

3. (10 pts) Suppose the government spends \$1 million on highways. Some of this money is earned by highway workers who in turn spend \$500,000 on food, travel, and entertainment. Workers in the food, travel, and entertainment industries then spend \$250,000 as a result of the highway workers' spending. The \$250,000 spending precipitates another round of spending, this time totaling \$125,000, and so on. Assuming that this process continues forever, what is the total spending generated by the initial \$1 million outlay? (Include the original \$1 million in your total.)

4. (a) (5 pts) Write down the first four nonzero terms in the Maclaurin series for $\cos(x)$.

(b) (5 pts) **Use part (a)** to write the first four nonzero terms in the Maclaurin series for $\cos(\sqrt{x})$. **Show your work!**

(c) (5 pts) To what number does the following series converge?

$$1 - \frac{2}{2!} + \frac{4}{4!} - \frac{8}{6!} + \frac{16}{8!} - \dots$$

5. A function $g$ has the Taylor approximation $g(x) \approx c_0 + c_1(x-a) + c_2(x-a)^2$, and the graph given below. What can you say about the signs of $c_0$, $c_1$, and $c_2$? Circle your answers **and justify your answers!**

(a) (5 pts) $c_0$ is negative zero positive.

(b) (5 pts) $c_1$ is negative zero positive.

(c) (5 pts) $c_2$ is negative zero positive.