Math 2254-05 Skills Test #2 February 16, 1999 key

Directions

You must show your work on this test paper. You may not use a calculator. There is no partial credit. There is no credit for answers without supporting argument. Your score on this test will **not** be dropped.

Each question is worth 10 points.

1. $\displaystyle\int \frac{5}{3x-2}\ dx$

   Let $u = 3x - 2$ so that $du = 3\ dx$ implying that $dx = 1/3\ du$ and $\displaystyle\int \frac{5}{3x-2}\ dx = 1/3\ \int \frac{5}{u}\ du = 5/3\ \int \frac{1}{u}\ du = 5/3\ \ln|u| + C = 5/3\ \ln|3x-2| + C$.

2. $\displaystyle\int x^3\, e^{x^4}\ dx$

   Let $u = x^4$ so that $du = 4\ x^3\ dx$, $dx = 1/4\ du$ and $\displaystyle\int x^3\, e^{x^4}\ dx = \int e^u\ 1/4 \cdot du = 1/4\ \int e^u du = 1/4\ e^u + C = 1/4\ e^{x^4} + C$.

3. $\displaystyle\int \frac{dx}{x^2+3x-10}$

   Note that $x^2 + 3x - 10 = (x+5)(x-2)$ so we can use partial fractions decomposition on the integrand. We must find constants A and B so that

$$\frac{1}{x^2+3x-10} = \frac{1}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$

   If we add the rational functions on the right hand side of the equation we have

$$\frac{A\ (x-2) + B\ (x+5)}{(x+5)(x-2)} = \frac{Ax - 2A + Bx + 5B}{x^2+3x-10} = \frac{-2A + 5B + x\ (A+B)}{x^2+3x-10}$$

   which must be equal to the integrand, $\displaystyle\frac{1}{x^2+3x-10}$. It follows that $1 = -2A + 5B + x\ \ (A+B)$. Since that must be true for all x, we have $1 = -2A + 5B$ and $0 = A + B$. Thus, $A = -B$ and $7B = 1$ implying $B = 1/7$, $A = -1/7$. Now we can rewrite the integrand using

$$\frac{1}{x^2+3x-10} = \frac{-1/7}{x+5} + \frac{1/7}{x-2}$$

   That lets us say

$$\int \frac{dx}{x^2+3x-10} = -1/7\ \int \frac{dx}{x+5} + 1/7\ \frac{dx}{x-2}$$

thus

$$\int \frac{dx}{x^2+3x-10} = -1/7\ \ln|x+5| + 1/7\ \ln|x-2| + C = 1/7\ ln\left|\frac{x-2}{x+5}\right| + C$$

4. $\int x\sqrt{3x^2+5}\ dx$

Use substitution. Let $u = 3x^2+5$ so that $du = 6x\ dx$ and $x\ dx = 1/6\ du$. This gives us $\int x\sqrt{3x^2+5}\ dx = \int u^{1/2}\ 1/6\ du = 1/6\int u^{1/2}\ du = 1/6\ \frac{u^{3/2}}{3/2} + C = 1/9\ u^{3/2} + C = 1/9\ (3x^2+5)^{3/2} + C$.

5. $\int \frac{\sin(1/x)}{x^2}\ dx$

Let $u = 1/x$ so that $du = -1/x^2\ dx$ and $dx/x^2 = -du$. This gives us $\int \frac{\sin(1/x)}{x^2}\ dx = -\int \sin u\ du = \cos u + C = \cos(1/x) + C$.

6. $\int 4e^{6x}\ dx$

Let $u = 6x$ so that $du = 6\ dx$ and $dx = 1/6\ du$. This gives us $\int 4e^{6x}\ dx = 1/6\ \int 4e^u\ du = 4/6\ \int e^u\ du = 2/3\ \int e^u\ du = 2/3\ e^u + C = 2/3\ e^{6x} + C$.

7. $\int x\ \ln(x)\ dx$

Use Integration by Parts. Let $u = \ln x$ so that $dv = x\ dx$. This gives us $du = dx/x$ and $v = x^2/2$. Then $\int x\ \ln(x)\ dx = (x^2/2)\ \ln x - \int (x^2/2x)\ dx = (x^2/2)\ \ln x - \int (x/2)\ dx = (x^2/2)\ \ln x - (x^2/4)\ + C$.

8. $\int \frac{x}{x^2+1}\ dx$

Use substitution. Let $u = x^2+1$ so that $du = 2x\ dx$ and $x\ dx = 1/2\ du$. This gives us $\int \frac{x}{x^2+1}\ dx = 1/2\int \frac{du}{u} = 1/2\ \ln|u| + C = 1/2\ \ln(x^2+1) + C$.

9. $\int \sqrt{2x-3}\ dx$

Use substitution. Let $u = 2x-3$ so that $du = 2\ dx$ and $dx = 1/2\ du$. Then $\int \sqrt{2x-3}\ dx = 1/2\int u^{1/2}\ du = 1/2\ \frac{u^{3/2}}{3/2} + C = 1/3\ (2x-3)^{3/2} + C$.

10. $\int x\cos(x)\ dx$

Use Integration by Parts. Let $u = x$ so that $dv = \cos(x)\ dx$. Then $du = dx$ and $v = \sin(x)$. This gives us $\int x\cos(x)\ dx = x\sin(x) - \int \sin(x)\ dx = x\sin(x) + \cos(x) + C$.