KEY

Math 2254-05 Test #1 February 8, 1999

Directions

You must show your work on this test paper. Do not use scrap paper. When you use your calculator, indicate **how** you use it, e.g., "I used my calculator to find $y' = 2^x$."

1. (5 pts) Find $\frac{d}{dx}(\sin^{-1}(x^2y))$. An answer alone is sufEcient.

$2xy / \sqrt{1-x^4y^2}$

2. **(7 pts) Let $\frac{dQ}{dt} = 300 - 0.3Q$. Solve the differential equation subject to $Q(0) = 500$.**

Using the calculator, $Q = C(0.740818)^t + 1000$ so

$Q(0) = 500 = C \cdot 1 + 1000 \Rightarrow C = -500$ so

3. **(8 pts) Let $f(x) = \frac{3x-7}{5x+11}$.** Use **calculus** to argue that **f** is invertible.

By the calculator $f'(x) = 68/(5x+11)^2 > 0$ for all $x \neq -11/5$ so f is increasing on its domain thus is 1-1, thus is invertible.

4. (7 pts) Find the inverse of f(z) = $(1.05)^x$.

Solve $x = (1.05)^y$ for y: $y = \ln(x^{20.4959})$

$y = 20.4959 \ln x$

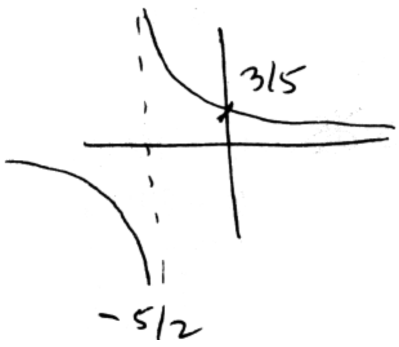
5. (8 pts) Let f(z) = $\sqrt{12x+13}$. Find g'(23) if g = f^{-1}.

$g'(23) = \frac{1}{f'(y)} = 23/6$ where $g(23) = y \Rightarrow 23 = \cancel{f(y)}$

$= \sqrt{12y+13}$

$\Rightarrow y = 43$

6. **(10 pts) Sketch the function which is the inverse of $f(x) = 3/(2x+5)$.**
Label intercepts and give equations for asymptotes.



invert

7. (20 pts) **Match the following differential equations with their solutions. Note: the given functions may satisfy more than one equation or none and some equations may have more than one solution or no solution.**

IV (a) $y'' = y$ (I) $y = \cos x$

no solution on the list ← (b) $y' = -y$ (II) $y = \sin(2x)$

V (c) $y' = 1/Y$ (III) $y = x^2$

I (d) $Y'' = -Y$ (IV) $y = e^x + e^{-x}$

no soln. ← (e) $y'' - 4y = 0$ (V) $y = \sqrt{2x}$

8. **An exponentially decaying substance was weighed every hour and the results are given below.**

Time	Weight (in grams)
9 am	10.000 → Q_0
10 am	8.958
11am	8.025
12noon	7.189
	6.440

(a) (7 pts) **Determine a formula of the form $Q = Q_0e^{-kt}$ which would give the weight of the substance, Q, at time t, where t is measured in hours since 9 am. Use your calculator, but indicate how you get to your answers. Give your numbers to three decimal places.**

$8.958 = 10e^{-k \cdot 1} \Rightarrow k = 0.110$ $\boxed{Q = 10e^{-0.110t}}$

(b) (8 pts) **Find the half-life of this substance, accurate to three decimal places.**

$5 = 10e^{-0.110t} \Rightarrow t = 6.299$ hrs.

9. **The population of a species of elk on a remote island has been monitored for some time. When the population was 600, the relative birth rate was 35% and the relative death rate was 15%. When the population was 800, the relative birth rate was 30% and the relative death rate was 20%. Assume there are no hunters and that there is no migration. Also assume that the relative growth rate (r.g.r.) is a linear function of the size of the population.**

(a) (10 pts) **Write a differential equation to model the population as a function of time. (Hint: Recal that r.g.r. = $k - aP$ and that the logistic model is $\frac{1}{P}\frac{dP}{dt} = k - aP$.)**

$0.35 - 0.15 = $ r.g.r

$\frac{1}{P}\frac{dP}{dt} = k - aP \Rightarrow$ $\cancel{\frac{1}{600}\frac{dP}{dt}} = k - 600a = 0.20$

$k - 800a = 0.10$

$200a = 0.1$

$\Rightarrow a = 1/2000$

$$k-(600)\left(\frac{1}{2000}\right)=0.2 \Rightarrow k=0.5$$

So the diff. eq. is

$$\frac{1}{P}\frac{dP}{dt}=0.5-0.0005P$$

(b) (5 pts) We say that the population P has reached the carrying capacity of the environment when the population can grow no further, i.e., when $\frac{dP}{dt}=0$. This is the largest population the environment can support. Fiid the largest population of elk the environment can support on this remote island.

$$\frac{1}{P}\cdot 0 = 0.5-0.0005P \Rightarrow \frac{0.5}{.0005}=\cancel{0.0005}P$$

$$\Rightarrow P=1000$$

(c) (5 pts) Sketch a rough graph of the population of elk as a function of time. How does the number you found in part (b) above figure into your sketch?