

KEY

Math 2254-05 Test #5 April 27, 1999

Directions

You must show your work on this test paper. Do not use scrap paper.

1. Determine which of the following converge and which diverge. For those which converge, determine the limit of the sequence. Show your work.

(a) (5 pts) $\left\{ \left(\frac{1}{2} \right)^n \right\}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{2} \right)^n = 0 // \text{ (Sequence converges to 0)}$$

(b) (5 pts) $\{ \sin(n) \}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \sin(n) \text{ does not exist (Sequence Diverges)}$$

(c) (5 pts) $\{ 1 + (-1/3)^n \}_{n=1}^{\infty}$

$$\lim_{n \rightarrow \infty} \left[1 + \left(-\frac{1}{3} \right)^n \right] = 1 // \text{ (Sequence converges to 1)}$$

(d) (5 pts) $\left\{ \frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \dots \right\}$ $a_n = \frac{n+1}{n}, n=1, 2, \dots$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right) = 1 // \text{ (Sequence converges to 1)}$$

2. Determine which of the following converge and which diverge. For those which converge, determine the exact limit of the series. Show your work.

(a) (5 pts) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$

$$\text{Geometric with } a=1, r=\frac{1}{2}.$$

$$\text{Converges to } \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2 //$$

(b) (5 pts) $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \dots$ $a_n = \frac{n+1}{n}$, $n=1, 2, \dots$
 $\lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} (1 + \frac{1}{n}) = 1 \neq 0$, series diverges

(c) (5 pts) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$: Harmonic Series, $a_n = \frac{1}{n}$
 The series diverges as shown in class.

(d) (5 pts) $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \frac{4}{81} + \dots$
 Geometric, $a = 4$, $r = -\frac{1}{3}$, converges to $\frac{4}{1 - (-\frac{1}{3})} = 3$

(e) (5 pts) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \dots$ Recall: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$
 This series converges to $\sin 1 \approx 0.841471$.

3. (10 pts) Suppose the government spends \$1 million on highways. Some of this money is earned by highway workers who in turn spend \$500,000 on food, travel, and entertainment. Workers in the food, travel, and entertainment industries then spend \$250,000 as a result of the highway workers' spending. The \$250,000 spending precipitates another round of spending, this time totaling \$125,000, and so on. Assuming that this process continues forever, what is the total spending generated by the initial \$1 million outlay? (Include the original \$1 million in your total.)

Total spending (in millions of dollars) is given by:
 $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \frac{1}{1 - \frac{1}{2}} = \underline{\underline{2 \text{ million dollars}}}$.

4. (a) (5 pts) Write down the first four nonzero terms in the Maclaurin series for $\cos(x)$.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$2!$ $\frac{(1-x)^2}{4!}$ $6!$

does the following series converge?

Reason: $g(a) > 0$. (See graph)