Some Advice on TI-89 Functions for Calculus I

By The Calculus Committee '99-'00

General Remarks

The Catalog key provides the user with a list of all built-in functions on the TI89, complete with instructions. (Look at the very bottom of the screen when you get to the function you want.) When you press the Catalog key, you put the keyboard into Alpha mode. To find a function you want after pressing Catalog, for example Limit, press the key corresponding to the first letter of the function. In our example, we would press 4 and that would put us at the top of the list of functions that start with the letter L.

If you are in the midst of using an unfamiliar function, and you need a reminder as to how to use the function, press the Catalog key again, scroll to the function, look at the bottom of the screen, then hit ESC.

You may access functions via menus, the Catalog, or by typing them in by hand.

Clear everything before you start on a new problem. A way to do that is to clear the command line (where you input commands), then access the NewProblem function (F6, 2, Enter). This clears all single letter entries in memory so that if you talk to the calculator about a, it will not know what you mean until you give it instructions as to how to interpret a. NewProb does not clear the memory of anything but the history (the last 30 lines) and single letter assignments. It does deselect what is on the current Y= menu, but it does not erase the functions on your list. (If you are in Function mode, for example, any selected functions on your Parametric or Polar lists are unaffected.)

Failure to start with NewProblem is a common source of errors. Get into the habit of using NewProblem right away and remind students of it early and often.

Do not use implicit multiplication with the TI89. If you try talking to it about $2\sin(x)$, for example, it will think that $2\sin$ is a variable name. You must type $2 \times \sin(x)$ if you mean $2\sin(x)$. Get into that habit right away and remind students of it early and often.

Use Second, scroll down arrow, to scroll to the top of the next screen. Use green diamond, down arrow, to get to the bottom of a list. These work when you are scrolling through the catalog, a menu, or the history screen.

The TI-89 has many editing features we use on the computer. The arrow key next the the Second key is for upper case. Using it with the scrolling keys allow you to highlight so that you can then cut and paste.

P.1-P.3

Calculator functions that come up in these sections are: graphing, including defining and graphing piecewise functions, negotiating the graph of an equation, evaluating functions, composing functions. Note the potential use of ComDenom when using the calculator to simplify difference quotients.

P1 Graphs, Equations

In this section, we should teach students how to graph an equation like $x^2 - 4x - y^2 = -3$ by asking the calculator to solve the equation for y and then using the two answers to define two functions. One can also do implicit graphing on the TI89. We do not recommend doing that the first week of the course. There are later opportunities to bring it up and the reader can refer to the notes here dealing with Chapter 2 to read more about using 3D Plot to get implicit graphs.

P2 Lines

Our recommendation is that students be able to deal with all the usual sorts of line problems without the calculator.

P3 Functions

Here we should show students how to graph piecewise functions using the *when(* command. The syntax is best illustrated with an example. To enter

$$f(x) = \begin{cases} 1 - x^2 \text{ if } x \leq 1 \\ x + 1 \text{ otherwise} \end{cases}$$

type when($x \leq 1$, $1 - x^2$, $x + 1$). This command can be nested to define functions with more than two pieces.

Note that you can access $\leq$, $\geq$, and $\neq$ respectively via green diamond <, green diamond >, green diamond =.

Demonstrate the Table feature of the calculator for the students. (It is very much like that on the TI82.)

Show the students that the calculator will find $f(a+h)$ if it knows what $f(x)$ is. (To make function assignments, for example $f(x) = \sqrt{2x+1}$, you can use the command, "Define $f(x) = \sqrt{2 \times x + 1}$.") Show them how to compose functions in general with the machine, via substitutions. (Define $f(x)$ as above, then define $g(x)$ to be some other function and ask the machine for $f(g(x))$.)

Talk to them about the calculator connecting dots and what that does to vertical asymptotes and points of discontinuity in general. This of course comes up when we graph piecewise functions. The students should know enough to graph with the Dot style in those cases. (Pick your graphing style from the Y= screen by selecting F6.)

Insist that students understand the basic types of transformations in the blue box on page 24 and that they be able to cope with them by hand as in problems 36-38. (This is in keeping with what the Precalculus Committee is recommending.)

Students should be able to do easy substitutions into functions by hand, for example, "If $f(x) = x^2$ what is $f(2x)$?" By the same token, they should be held responsible for knowing how to use the TI-89 for entering piecewise functions, using the table feature, and evaluating and composing functions.

Chapter 1

The main objective of the first real chapter in the book is to establish limits and continuity. The text emphasizes numerical and graphical analysis.

Section 1.1 Preview of Calculus

Here we could show the students how the TI-89 will find the distance between points on a graph and how it will draw secants on curves by connecting points. From the graph screen, with a curve in the picture, hit F5, selection 9. You can use arrows to select your points or you can type the x-coordinates. This feature gives you distance and the secant in one shot.

Show them how to use the list feature for graphing on the TI-89: it works exactly as illustrated in problem number 9 on page 46.

Section 1.2 Limits Graphically and Numerically

The table feature on the TI-89 comes in handy in this section. Emphasize here what graphing calculators will and will not show you vis a vis limits. (The book does a good job of that itself.)

Section 1.3 Limits Analytically

Students should be held responsible for knowing how to use the limit function on their calculator.

Section 1.4 Continuity and One-Sided Limits; Intermediate Value Theorem

The following example gives the syntax for one-sided limits: limit(1/x,x,0,-1) gives $\lim_{x\to 0^-} \frac{1}{x}$. What is not obvious is that for *any* left hand limit, the last optional entry in the limit command should be -1 (any negative number will do) and for *any* right hand limit, the last entry should be 1 (any positive number will do.) **Using numbers to the left and right of the point where you take the limit will not work!** Try it on $\lim_{x\to 5^+} \frac{1}{(x-5)}$ to see.

Students should be held responsible for using the table feature and for using graphs to recognize situations where the limits do not exist. The graphing and table features on the calculator can help them with one-sided limits, as well.

The greatest integer function on the TI-89 is floor(. It comes up in examples and problems in this section.

Section 1.5 Infinite Limits

Students should know that when the downstairs of a rational function is zero, there might be a vertical asymptote. They should exercise appropriate skepticism regarding the calculator's inability (when graphing in Line style) to recognize a vertical asymptote. They should understand the features of a graph that suggest a function goes to infinity or negative infinity as $x \to a$.

Testing With the TI-89

We recommend testing in two pieces: one without the calculator, the other with the calculator. On the no calculator piece, one might make the questions very simple and straightforward but offer nothing in the way of mercy points or partial credit. On the part allowing for calculator, ask for more than you would without the calculator.

One might compose a test using a selection of problems like these.

Sample Test Questions for Test #1

Do the following problems without your calculator. There is no partial credit.

1. Find the intercepts for the graph of $y = 2x - 3$.
2. Sketch the graph of $3x - 4y = 2$. Label intercepts.
3. If $f(x) = 1/x$, find $f(x + h)$.
4. The graph of $f(x)$ is given below. Use it to sketch $f(x - 2)$.
5. Is the following statement true or false? "If $f(x)$ is an even function and $(-1, 2)$ is on the graph of $y = f(x)$ then $(-1, -2)$ is also on the graph of $y = f(x)$."
6. Sketch $y = \sqrt{9 - x^2}$. Label intercepts.
7. What is the domain of $f(x) = \sqrt{3x - 4}$?
8. If $f(x) = \cos(x)$ and $g(x) = x^2$, what is $f \circ g(x)$?
9. Find $\lim_{x\to 1} 1/(x - 1)^2$.

10. Find $\lim_{x\to\pi/2} \sin(x)$.

11. Find $\lim_{x\to 1+} 1/(x-1)$.

12. Does $f(x) = 1/(x-2)$ have any vertical asymptotes? If so, where?

13. Use the following graph (picture as on page 76, top) to find $\lim_{x\to c+} f(x)$, $\lim_{x\to c-} f(x)$, $\lim_{x\to c} f(x)$.

You may use your calculator to do the following but you must argue in support of your answers, even on true/false questions. Valid arguments include algebraic manipulations, graphs, tables, and theorems. For true/false questions, argue why the true statements are true and give examples to show the false statements are false.

1. Discuss the symmetry of $y = x^3 + x$ with respect to both axes and the origin.

2. Write a function that has a graph symmetric with respect to the y-axis and an intercept at $x = -1/3$.

3. If $f(x) = 5x^3$, find and simplify $\frac{f(x+h) - f(x)}{h}$.

4. Show that $f(x) = a_0 + a_1x^2 + a_2x^4 + ... + a_nx^{2n}$ is even.

5. True or false? If f is a function then $f(ax) = af(x)$.

6. In one picture, sketch $y1(x) = 4x - x^2$ and $y2(x) = \frac{y1(1 + \{1, 0.5, 0.25, 0.1\}) - y1(1)}{\{1, 0.5, 0.25, 0.1\}}(x-1) + y1(1)$. Give a written description of the graphs of $y2$ relative to $y1$. Use the graphs of $y2$ to estimate the slope of $y1$ at $(1, 3)$. To improve your approximation of the slope, how could you change the list in the formula for $y2$?

7. Write a brief description of the meaning of the following mathematical statement: $\lim_{x\to 1} f(x) = 7$.

8. What is $\lim_{x\to 0} \frac{\sin(2x)}{x}$?

9. True or false? If the limit of $f(x)$ as x approaches c is L, then $f(c) = L$.

10. Suppose $f(x) = \begin{cases} x^2 + 2 & \text{when} \quad x \le 1 \\ 2x - 3 & \text{when} \quad x > 1 \end{cases}$. Find $\lim_{x\to 1} f(x)$.

11. Graph $f(x) = x\sin(1/x)$, $y = |x|$ and $y = -|x|$ all on the same set of axes. Now find $\lim_{x\to 0} f(x)$ and use the Squeeze Theorem to justify your answer.

12. Find a constant a so that $f(x) = \begin{cases} x^4 - 2 & \text{when} \quad x \le 2 \\ ax^2 & \text{when} \quad x > 2 \end{cases}$, is continuous on the whole real line.

13. Let $f(x) = \begin{cases} -3x + 4 & \text{when} \quad x < 1 \\ x^2 & \text{when} \quad x \ge 1 \end{cases}$. Find the x-values (if any) where f is discontinuous. Which discontinuities are removable?

14. From the graph of the function $f(x) = \frac{|x^2 + 4x| * (x+2)}{(x+4)}$, estimate $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$. Is the function continuous on the whole real line?

15. Explain how the Intermediate Value Theorem guarantees the existence of a zero for $f(x) = 1 + x - 3\tan(x)$ on the interval $[0, 1]$.

16. True or false? If $f(x) = g(x)$ for $x \neq c$ and $f(c) \neq g(c)$ then either f or g is not continuous at c.

17. Find $\lim_{x \to 0^-} (x^2 - 1/x)$.

18. Determine whether the following function has a vertical asymptote or a removable discontinuity at $x = 2$.

$$f(x) = \frac{x^2 - 3x + 2}{x^2 - x - 2}$$

19. True or false? Polynomials have no vertical asymptotes.

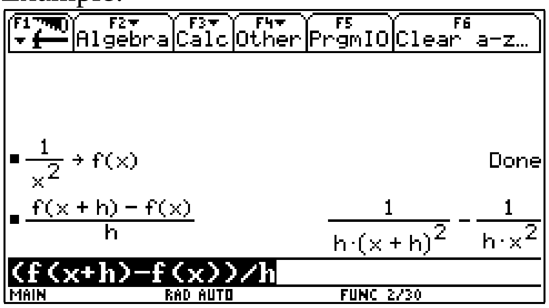
A Note on Chapter 2

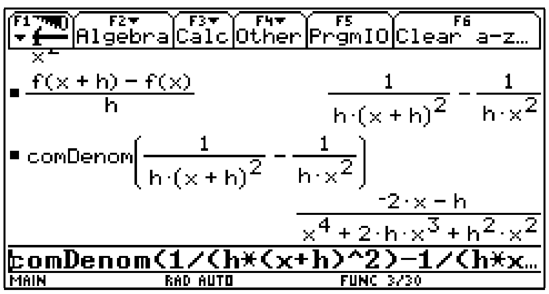
The avgRC function on the calculator finds the average rate of change of a function on the interval $(x, x+0.001)$. By inserting the last (optional) argument as h, or anything else you want, you can force it to find the average rate of change of your function on your favorite interval, including $(x, x+h)$. Some instructors may want to take advantage of this function during the discussion of average rates of change vs. instantaneous rates of change that come up in Chapter 2. Also, using the "with" key, i.e., the vertical bar just below the key for =, you can evaluate the average rate of change of a function $f(x)$ on an interval such as $(5, 5+h)$. Use the following keystrokes: avgRC$(1/x,\ x,\ h)|x = -1$, to see an example.

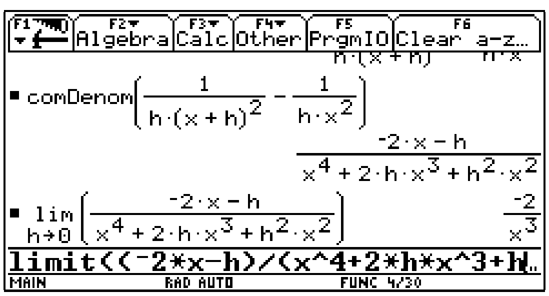
Section 2.1:

Students should be able to find the derivatives of simple functions (See the “NC” problems assigned in this section) by hand using the definition. Others can be done with the aid of the calculator. Use ComDenom as necessary with difference quotients. Also, use the symbol “h” rather than “Δ x” in difference quotients, because the latter is a reserved variable name.

Example:

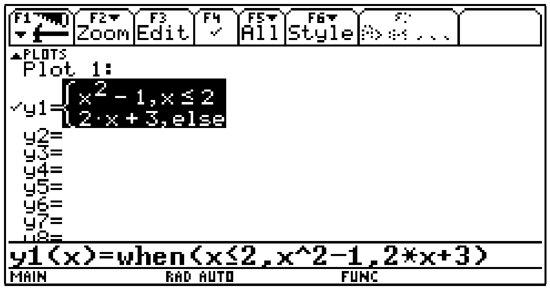


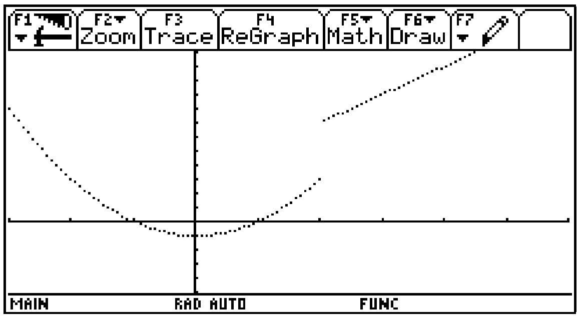




Students should understand the connection between the graph and differentiability. Graphs of simple functions (e. g. 27-32) should be sketched by hand. More complicated ones should be done using the machine.

Example: A Piecewise Defined Function

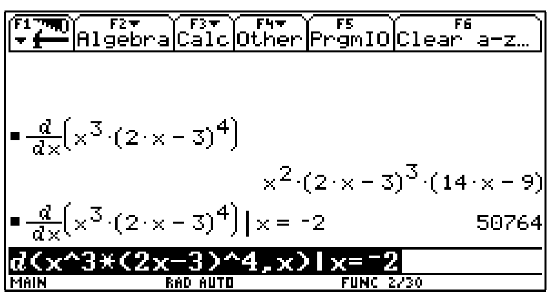


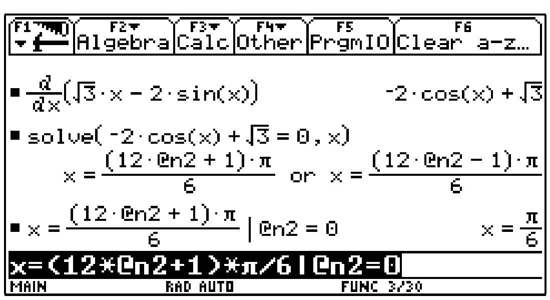


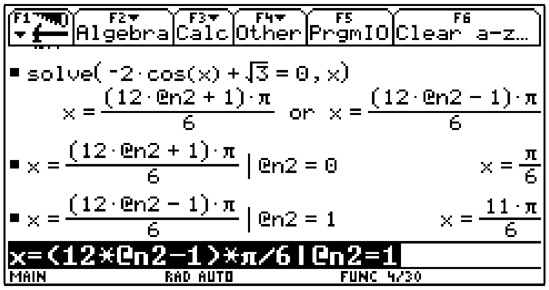
Section 2.2:

Students should be able to find the vast majority of derivatives in this section by hand, and perhaps check their work with the calculator.

Examples:



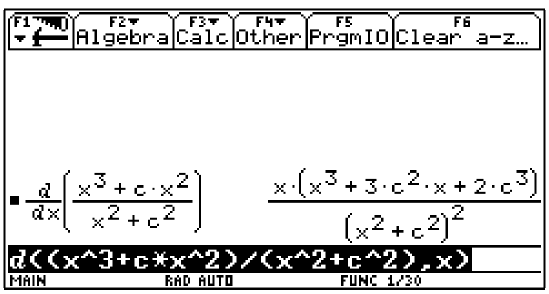


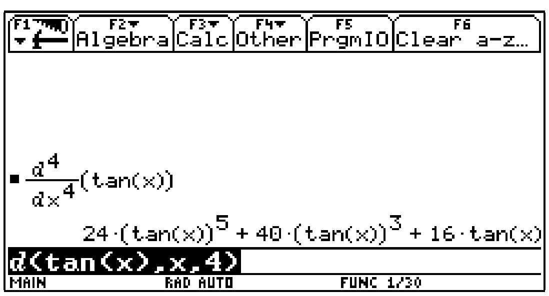


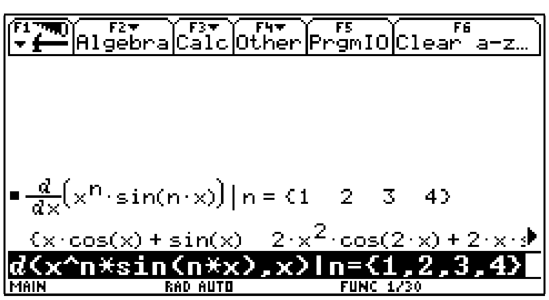
Section 2.3:

This section covers the product and quotient rules as well as higher derivatives. Most differentiation should be accomplished by hand and checked with the machine. Really messy derivatives or higher derivatives should be done on the machine.

Examples:

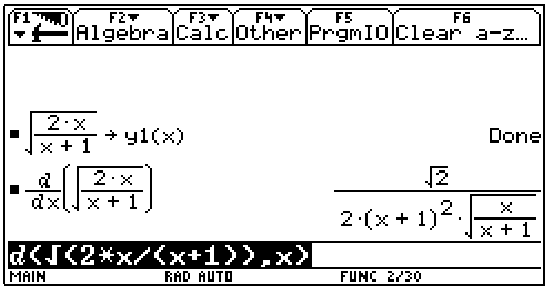


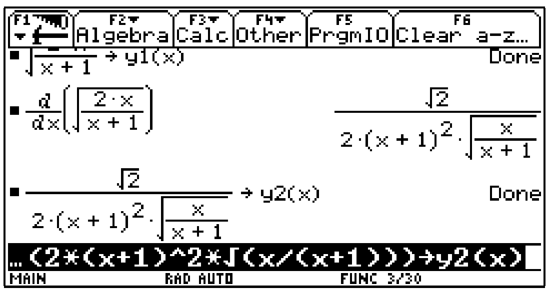


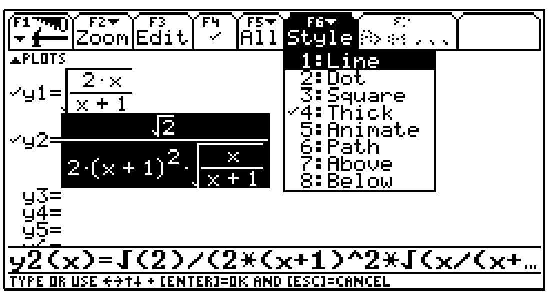


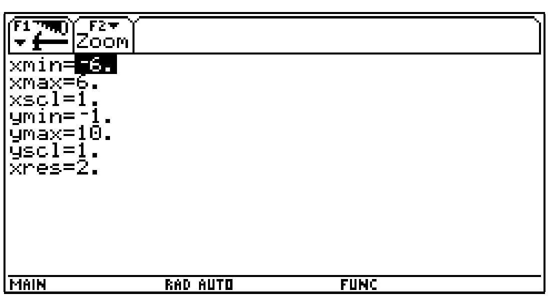
Section 2.4:

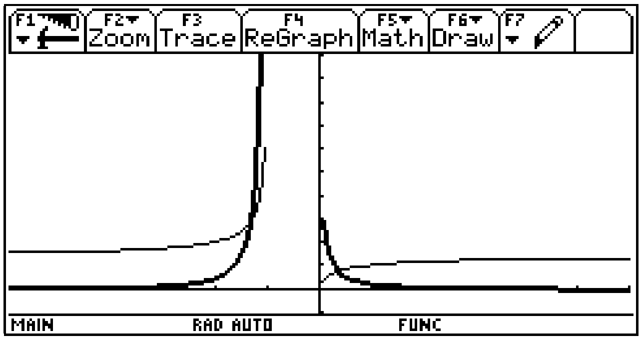
This section discusses the chain rule. Most problems should be done by hand and checked by machine. When graphs are requested, or in certain application problems, the calculator may be used.







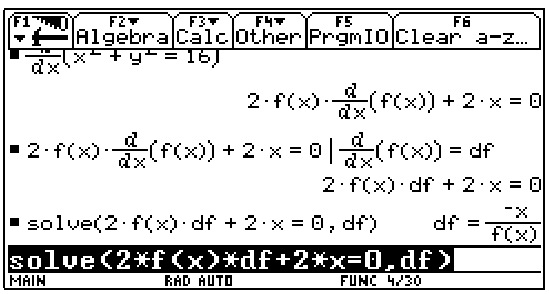


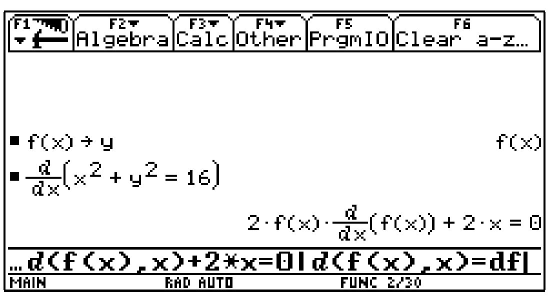


Section 2.5:

This section covers implicit differentiation. The assigned problems give some idea of the level of pencil and paper computational skill expected of students. The TI-89 does not have built-in implicit differentiation. However, there are ways to get the job done with the aid of the calculator.

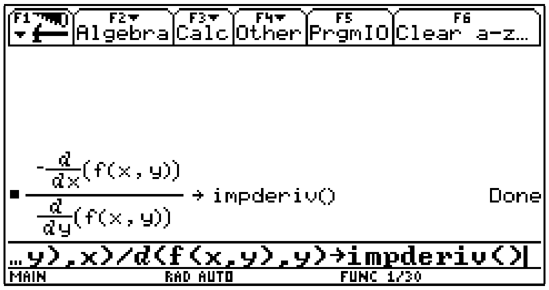
Example 1: Using the procedure of implicit differentiation and the TI-89:

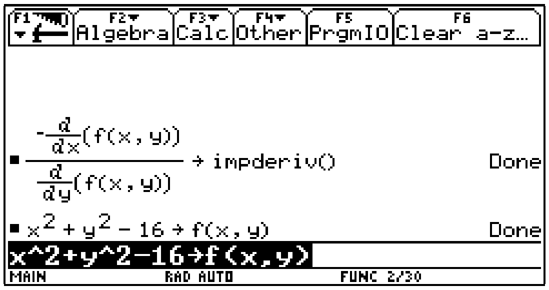




Of course, we must interpret the last screen correctly and conclude that dy/dx = -x/y .

Example 2 : Creating a user-defined function:

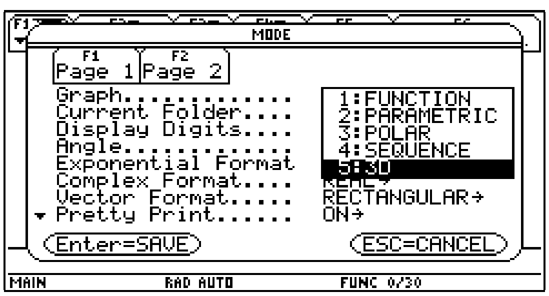


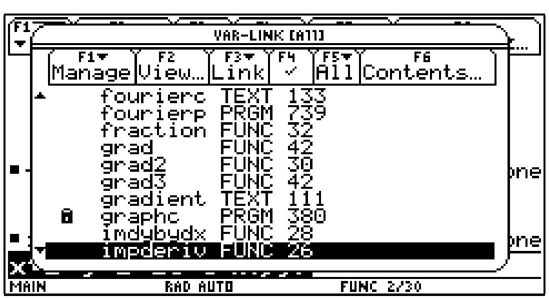


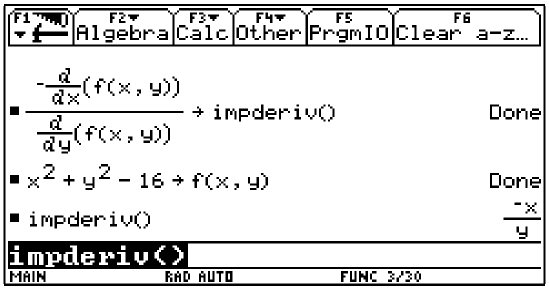
The TI-89 will graph implicitly defined functions. You may use the procedure explained in the next example. Due to software limitations, I am unable to show you the final graph, because I am using screen captures from a TI-92 rather than an 89. Below is the best I can do subject to this constraint.

Graphing Implicitly Defined Functions:

Step 1: Set the Graph Mode to 3-D:





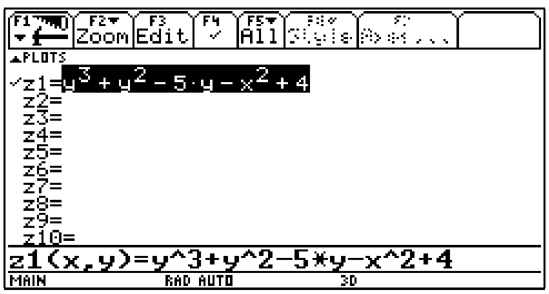


Step 2: Enter your function into the Y = Editor. For example:

y3 + y2 - 5y - x2 = -4

would be entered as:

z1 = y^3 + y^2 - 5*y -x^2 + 4



Step 3: Set the graphing window. For graphing implicitly defined functions always set
eyeθ = -90, eyeϕ = 0, and eyeψ = 0.
Now, for this particular example, choose: xmin = -4, xmax = 4, xgrid = 14, ymin = -4, ymax = -3, ygrid = 14, zmin = -5, zmax = 5.

Step 4: From the Y= editor, press F1 (Tools), 9 (Format), and set Coordinates to RECT, Axes to AXES, Lables to ON and Style to IMPLICIT PLOT. Now Graph. You must be patient because the calculator takes a while to display the graph. When it does display, you will be looking straight down the z-axis into the xy-plane. See Figure 2.25 in the text for the correct graph.

The Rest of the Course

At this stage in the course, most instructors will have sufficient command of the calculator to be able to access new functions and make use of them appropriately. The suggested pool of problems provides quite a bit of guidance in that direction as well. The rest of this document spotlights a few places where new techniques or calculator functions can be brought to bear.

The Committee's guiding principle on calculator usage at this stage can be summarized as follows: for easy calculations, done expressly for the purpose of familiarizing students with the mechanics of algebra/calculus, pencil and paper are appropriate. For calculations that arise in the course of challenging applications, even in cases where the calculations are not tremendously challenging, the Committee recommends allowing use of the calculator. We feel that this allows the students to direct their attention to the set-up, the more demanding part of the problem, and the one where calculators and computers are no use.

We also recommend using the calculator in the classroom for explorations of the "what if..." variety.

Chapter 3

One can use the calculator to find critical points by invoking combinations of the derivative function and the solve function.

The propfrac function will rewrite an "improper" rational function so that a slant asymptote becomes obvious.

The syntax solve($f(x) = f(-x)$, x) can test for symmetry.

The spreadsheet that follows summarizes the Committee's recommendations on where to draw the line between pencil and paper calculations and machine calculations for the algebraic functions that occupy so much of this chapter.

	Nice poly. x^3-3x^2+3	Nasty Poly. $x^4-5x^3+12x^2+1$	Nice Rational $\frac{2(x^2-9)}{(x-2)(x-4)}$	Nasty Rational $\frac{x^3+x+1}{x^2+2x-2}$	Nice Radical $\frac{x}{\sqrt{x^2+2}}$	Nasty Radical $2x^{5/3}-5x^{4/3}$
Domain	NC	NC	NC	C	NC	NC
Range	C	C	C	C	C	C
Derviatives	NC	NC	C	C	C	C
x-intercepts	NC	C	NC	C	NC	C
y-intercept	NC	NC	NC	NC	NC	NC
vertical asymptotes	N/A	N/A	NC	C	NC	C
horizontal asymptotes	N/A	N/A	NC	NC	NC	C
critical points	NC	C	C	C	C	C
possible inflection pts.	NC	C	C	C	C	C
inflection points	NC	C	C	C	C	C
symmetry	NC	NC	C	C	C	C
increasing/decreasing	NC	C	C	C	C	C
concavity	NC	C	C	C	C	C
extrema	NC	C	C	C	C	C

	Nice Trig $\sin(x)+2\cos(x)\sin(x)$	Nasty Trig $\sin(x)+\sin(3x)$
Domain	NC	NC
Range	C	C
Derviatives	NC	NC
x-intercepts	NC	C
y-intercept	NC	NC
vertical asymptotes	N/A	N/A
horizontal asymptotes	N/A	N/A
critical points	NC	C
possible inflection pts.	NC	C
inflection points	NC	C
symmetry	C	C
increasing/decreasing	NC	C
concavity	NC	C
extrema	NC	C

Section 3.7: Optimization

In the simplest problems, we might ask students to go from start to finish. We also recommend testing in two parts on this material. On some problems, ask for the objective function and nothing further. For these problems, allow no calculator as many students use the calculator for a crib sheet. In other problems, give the objective function and have the students do the follow through, usually with the calculator. In the latter type of problems, we are suggesting that instructors allow the students free use of the calculator, even for finding derivatives.

Section 3.9: Differentials

Problems such as number 7 can be worked on the calculator as follows:

$$1 \to x : 0.1 \to dx$$
$$(x + dx)^3 - x^3$$
$$3x^2\ dx$$

Chapter 4

Section 4.1 Antiderivatives

For checking, students could graph families of antiderivatives like this: Graph $\int(f(x)\ dx) + \{-2, -1, 0, 1, 2, 3\}$.

Section 4.2 Area

For problems 41-47, ask the calculator for the sum from 1 to n. Then take the limit by hand (or machine). The summing command is "sum." Alternatively, calculate the sum for large values of n only and use that to argue what the limit is.

Chapter 6

Here the Committee's suggestion applies to insist that students be able to set up any problem without the aid of their calculators and follow through using the calculator for absolutely everything.