Math 2254 Test # 4

1. Find the intervals of convergence, including endpoints.

a) $\sum_{n=1}^{\infty} \frac{(x-6)^n}{n}$ b) $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^3}$

2. Use the definition of Taylor series (or Taylor polynomial) to find the 3rd degree Taylor polynomial centered at -1 for $f(x) = (2+x)^{-3}$.

3. a) Use any method to calculate the first 4 **non-zero** terms of the Taylor series centered at 0 for

$$f(x) = \frac{1}{1+x^3}.$$

b) Use your answer in part a) to get 3 non-zero terms of the Taylor series centered at 0 for

$$f'(x) = -\frac{3x^2}{(1+x^3)^2}.$$

4. a) Use any method (including memory) to write down at least 5 **non-zero** terms of the Maclaurin series for sin x.
b) Determine how many terms are needed to calculate sin (0.5) to within 0.001.
c) Use as many terms as you decided in part b) to calculate an approximate value for sin (0.5). Give your final answer in decimal form.

5. Recall that the hyperbolic cosine function is defined by cosh x = $\frac{1}{2}\left(e^x + e^{-x}\right)$.

a) Use the Maclaurin series for e^x and e^{-x} to get the first 4 **non-zero** terms of the Maclaurin series for cosh x.
b) Use your answer in part a) to estimate cosh 3. Give your final answer in decimal form.
c) Do you think your estimate in part b) is good? Why or why not?

6. a) Use any method to get the first 4 terms of the Maclaurin series for $f(x) = \sqrt[4]{1+x}$.

b) Use your answer from part a) to estimate $\sqrt[4]{0.8}$. Note that this is $f(-0.2)$. Give your final answer in decimal form.

7. a) Use any method to find the Taylor series centered at 0 for $f(x) = x^2 \cos x$. Write out at least 3 **non-zero** terms, and also give the series in sigma notation.

b) Use the first 3 **non-zero** terms of your series in a) to estimate $\int_0^1 x^2 \cos x \, dx$. Give your final answer in decimal form.

c) Use one of the methods that we have studied to estimate the error of your answer to part b).