Math 2345 Test #1

1. Give an example of a statement that is a tautology. Use English or logical symbols.

2. a) Make a truth table for the statement $(\sim p \wedge q) \rightarrow \sim q$.

   b) If p is true and q is false, is the statement $(\sim p \wedge q) \rightarrow \sim q$ true or false?

3. Give the converse of the statement: If Fyodor is strong, then Fyodor is fearful.

4. Negate the statements. Write your answers in good English:
   a) Joe eats burgers and Jane does not like fish.
   b) If Sara is a survivor, then Sara is famous.
   c) Every hockey player is missing some teeth.
5. Rewrite the statement in good English with no mathematical symbols:
   $\forall$ real numbers x, $\exists z \in \mathbf{Z}$ such that z>x.

6. Rewrite the statement formally using quantifiers and variables:
   Every cloud has a silver lining.

7. Give a counterexample to show that the statement is false:
   If any 2 even integers are added together, then the sum of the 2 even integers is divisible by 4.

8. Write the first 4 terms of the sequence defined by $a_n = \frac{n+1}{2n-5}$

9. Write the sum using summation notation: $\frac{3}{4\cdot 5} + \frac{4}{5\cdot 6} + \frac{5}{6\cdot 7}$.

10. Compute the sum and simplify: $\sum_{k=2}^{4} (3k-2)$

11. Compute the product and simplify: $\prod_{k=1}^{4} \frac{k}{k+2}$.

12: Simplify: $\frac{(n-1)!}{n!}$, assuming n>0.

13. Give the contrapositive of the following statement:
    For every integer n, if n is even then n^2 is divisible by 4.

14. Give a direct proof of the statement: If n is any even integer, then $(-1)^n = 1$.

15. Give a direct proof of the statement: For all integers a, b, and c, if a|b and a|c, then a|(b-c).

16. Prove the statement using the method of mathematical induction:

    $1 + 3 + 5 + \cdots + (2n-1) = n^2$ for all integers n ≥ 1.

17. Prove the following statement **either** by contraposition or by contradiction. Indicate your choice of method:

    For all integers n, if n^2 is odd, then n is odd.