Math 2345 Test #2 Show all your work. For problems with numerical answers, give your finalanswer in ordinary decimal form.

1. Is $\varnothing = \{\varnothing\}$? Explain.

2. Let A = {1, 2, 5, 8}, B = {5, 8}, C = {1, 5, 9, 10}, and let the Universal set U be U = {1, 2, 3, . . . , 10}
   a) Is $A \subseteq B$? b) Is $B \subseteq C$?

   c) Find $A \cap C$ d) Find $B \cup C$ e) Find A^c f) Find $(A \cap C)^c$ g) Find $(A \cap B) \cap C$

   h) List the elements of BxA. i) List the elements of BxB.

3. Recall that a bit string is a sequence of 0's and 1's.
   a) How many bit strings are there of length 10?

   b) How many bit strings of length 10 start with 1 and end with 1?

   c) How many bit strings of length 10 have 7 0's and 3 1's?

4. Evaluate: a) P(12,2) b) P(234,1) c) $\binom{8}{3}$

5. a) How many ways can 8 people line up at the betting window at the racetrack?
   b) How many ways can the 8 people line up if two of them insist on standing next to each other?

6. How many integers from 1 through 1000 are multiples of 3 or multiples of 7?

7. How many non-negative integer solutions does the equation $x_1 + x_2 + x_3 + x_4 = 12$ have?

8. How many distinct arrangements are there of the letters of the word "VOODOODOG"?

9. A man owns 5 dogs and 4 cats.
   a) How many ways can he select 4 of his animals to take for a walk?
   b) How many ways can he select 4 animals to take for a walk, if 2 are cats and 2 are dogs?
   c) How many ways can he line all the animals up for their baths, if all the dogs must be together and all the cats must be together?

10. Let A = {1, 2, 3}. List all elements in P(A), the power set of A.

11. Let B be the set of 26 letters of the alphabet. How many subsets of size 8 does B have?

12. Let Z be the set of integers.
    Let $A = \{n \in Z \mid n = 3k \text{ for some integer } k\}$
    Let $B = \{n \in Z \mid n = 3k + 1 \text{ for some integer } k\}$
    Let $C = \{n \in Z \mid n = 3k + 2 \text{ for some integer } k\}$. Is {A, B, C} a partition of Z? Explain your answer.

13. A bin of 30 computer boards contains 4 defective. How many samples of 6 boards will contain at least one defective board?

14. Give a transition diagram for a finite state automaton that accepts strings of 0's and 1's that start with 10 and end with 11.

16. Prove the statement using the method of mathematical induction:

    $1 + 3 + 5 + \cdots + (2n-1) = n^2$ for all integers $n \geq 1$.