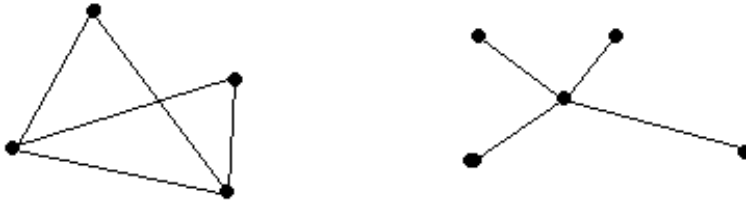


**Math 3345 Test # 3 Problems 1-3 are true/false.**

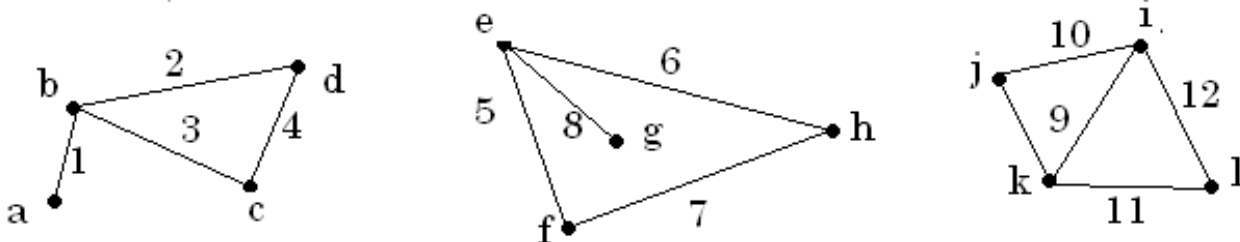
1. If graph  $G$  is isomorphic to graph  $H$ , and if  $G$  has exactly 5 vertices of index 4, then  $H$  must also have exactly 5 vertices of index 4.
2. A graph is a tree if and only if the graph has  $n$  vertices and  $n-1$  edges for some positive integer  $n$ .
3. Every connected graph has a spanning tree.
4. Give two of the isomorphism invariants of the pictured graph.



5. Either draw a graph with the specified properties or explain why it can't exist.

- a) A simple graph with 5 vertices of degrees 2, 3, 3, 3, 5.
- b) A graph with 4 vertices of degrees 4, 5, 7, 9.
- c) A graph with 5 vertices and total degree 8.
- d) A graph with 7 vertices and 6 edges that is not a tree.
- e) A tree with 6 vertices, each of degree 2.

6 Determine whether the graphs are isomorphic. If the graphs are isomorphic, indicate corresponding vertices given by an isomorphism of the graphs.



7. Draw all nonisomorphic simple graphs with 4 vertices.

8. In a group of 15 people, is it possible for each person to have exactly 3 friends among the other people of the group? Assume that friendship is a symmetric relation. Explain the reasoning for your answer.

9. Recall that a complete graph is a simple graph for which each pair of vertices has exactly one edge.

a) How many edges does a complete graph with 12 vertices have?

b) How many nonisomorphic complete graphs with 10 vertices are there?

10.  $S$  is a relation from  $A = \{1, 2, 3\}$  to  $B = \{4, 5, 6\}$  given by  $S = \{(1,5), (2,5), (3,4), (1,4)\}$ .

a) Give the inverse relation.

b) Is  $S$  a function? Why or why not?

11. Define a relation  $G$  on  $\mathbb{R}$  by  $aGb$  if  $a \geq b+1$ .

a) Is  $(5,2) \in G$ ?

b) Is  $(6,5) \in G$ ?

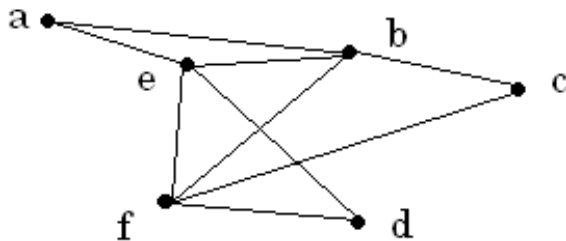
c) Is  $3G7$ ?

d) Draw the graph of  $G$  in the Cartesian plane.

12. Find 2 binary relations from  $\{a, b\}$  to  $\{1, 2\}$  that are not functions.

13. Let  $X$  be any non-empty set, and let  $P(X)$  be the power set of  $X$ . Define the relation  $\mathcal{S}$  on  $P(X)$  by  $A \mathcal{S} B$  if  $A \subseteq B$ .

Is  $\mathcal{S}$  reflexive, symmetric, transitive? Justify your answers.



14. For this graph, either describe an Euler circuit that starts and ends at vertex  $a$  or say why an Euler circuit does not exist.

15. What is the difference between a circuit and a simple circuit?