

Differential Equations Exam 1

June 1, 2007

Directions: Answer each question as completely as possible. Include all of your work and reasoning, as partial credit will be given on the basis of incomplete or partially correct answers. This means all justification should be included in your answers. Show all of the work on your own paper. Pay careful attention to be clear in your explanations.

1. A 100-volt electromotive force is applied to an RC series circuit in which the resistance is 200 ohms and the capacitance is 10^{-4} farad. Find the charge $q(t)$ on the capacitor if $q(0) = 0$. Find the current $i(t)$.

2. a) Give the order of the differential equation $x \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 12xy = 0$, and determine whether the differential equation is linear.

b) Show that $y = c_1 e^{-3x} + c_2 e^{-4x}$ a solution to the differential equation. Find the solution to the initial value problem given by the differential equation and the initial values $y(0) = 3$ and $y'(0) = -11$.

3. Find a general solution to the following two differential equations:

a) $\frac{dy}{dx} = (x^2 + 1)(y^2 + 1)$

b) $\frac{1}{x}y' + e^x y = 0$.

4. Find the solution to the initial value problem given by $y \frac{dy}{dx} - x = 2y^2$ and the condition $y(1) = 5$.

5. Solve the initial value problem given by $x^2 y' = y - xy$ and $y(-1) = -1$ using separation of variables.