

Differential Equations Exam 2

June 13, 2007

Directions: Answer each question as completely as possible. Include all of your work and reasoning, as partial credit will be given on the basis of incomplete or partially correct answers. This means all justification should be included in your answers. Show all of the work on your own paper. Pay careful attention to be clear in your explanations.

1.

a) Verify that the given two parameter family of functions is the general solution of the non-homogeneous differential equation $2x^2y'' + 5xy' + y = x^2 - x$. Include verification of homogeneous solutions, the particular solution, and that a fundamental set of solutions is included. The family is $y = c_1x^{-1} + c_2x^{-\frac{1}{2}} + \frac{1}{15}x^2 - \frac{1}{6}x$.

b) Are $f(x) = x$, $g(x) = x^2$, and $h(x) = 6x^2 - x$ linearly independent?

2. Find a second solution to the differential equation $xy'' + y' = 0$ using the methods of section 4.2. The given first solution is $y = \ln(x)$.

3. Solve the following two boundary value problems:

a) $y'' - 2y' + 2y = 0$, $y(0) = 1$, and $y(\pi) = 1$.

b) Same differential equation, with $y(0) = 1$ and $y(\frac{\pi}{2}) = 1$.

4. Find the particular solution to the differential equation given by $y'' + 2y' + y = e^{-x} + 3\cos(2x)$.

5. Find the general solution to the differential equation $4y'' + 36y = \csc(3x)$.