

Differential Equations Exam 3

July 2, 2007

Directions: Answer each question as completely as possible. Include all of your work and reasoning, as partial credit will be given on the basis of incomplete or partially correct answers. This means all justification should be included in your answers. Show all of the work on your own paper. Pay careful attention to be clear in your explanations.

1&2.

Solve the system of differential equations by systematic elimination.

$$Dx + D^2y = e^{3t}$$

$$(D + 1)x + (D - 1)y = 4e^{3t}.$$

Please solve for y first, and then x . Be sure to eliminate any unnecessary constants.

3. A 20 lb weight stretches a spring 6 inches. The weight is released from rest 6 inches below the equilibrium position. Find the position of the weight at $t = \frac{\pi}{4}$ s.

4. Find the charge $q(t)$ on the capacitor in an LRC series circuit when $L = 0.25h$, $R = 10$ ohms, $C = 0.001$ farad, $E(t) = 0$, $q(0) = 3$ C, and $i(0) = 0$. Use the provided equations to express your answer with only one trigonometric function.

5. A mass of 1 slug is attached to a long spring, stretching it 16 feet. The subsequent motion takes place in a medium that provides a damping force numerically equal to twice the instantaneous velocity. In addition, a fan alternately blows on and off the mass, providing an external force $f(t) = 4\cos(t) + 2\sin(t)$. The mass is released from equilibrium with a downward velocity of 3 feet per second.