

Differential Equations Exam 4

July 16, 2007

Directions: Answer each question as completely as possible. Include all of your work and reasoning, as partial credit will be given on the basis of incomplete or partially correct answers. This means all justification should be included in your answers. Show all of the work on your own paper. Pay careful attention to be clear in your explanations.

1. Use the definition to find the LaPlacian of $f(t)$ if:

$$f(t) = \begin{cases} 0, & \text{if } 0 \leq t \leq 1 \\ t, & \text{if } t \geq 1 \end{cases}$$

2. Use the LaPlace Transform to solve the initial value problem given by $y'' + 5y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

3. Find

- a) The inverse transform of $\frac{s}{s^2+4s+5}$
- b) The transform of $t^6 e^{-2t}$

4. Solve the system of differential equations for $y(t)$ only:

$$x' + y = t$$

$$4x + y' = 0, \text{ with } x(0) = 1 \text{ and } y(0) = 2.$$

5. Expand $f(x)$ in a Fourier Series.

$$f(x) = \begin{cases} 0, & \text{if } -\pi \leq x \leq 0 \\ \pi - x, & \text{if } 0 \leq x \leq \pi \end{cases}$$