Math and the GRE

Name

Date

Location

Test Format

- 30-question verbal section (30 min.)
- 28-question quantitative section (45 min.)
- Analytical writing in two tasks (75 min.)
- (possibly) experimental section
- (possibly) research section
- The experimental sections are not identified. You should treat them as part of the test.

The quantitative section

- The testing goal of this section is **not** to measure skills on advanced mathematical theory.
- The goal is to measure your ability to use and reason with numbers or mathematical concepts.
- The mathematical concepts are expected to be part of everyone's background.

The GRE is a CAT

- A Computer-Adaptive Test will present the taker with a randomly chosen question. Depending on how you answer each question, the computer will determine the next question offered.
- The algorithm the computer uses for offering subsequent questions is too complex to be worth learning as part of your study for the GRE.

Four useful true facts

- You must answer each question before moving on
- Once you have answered and confirmed, you can’t go back
- Hard questions are worth more points than easy ones
- The general GRE does not penalize for guessing

Quantitative Ability

- 14 quantitative comparison questions
- 10 discrete quantitative questions
- 4 data interpretation questions
- We'll look at them in an order today, but on the test day, they will be mixed up.
- First, we'll review general concepts that you can expect to appear in the questions.

All of the numbers used in the GRE are real numbers – this means there are no complex or imaginary numbers.

Numbers less than zero are called negative, and those greater than zero are called positive.

Negative numbers are always less than positive numbers. eg. $-3 < 3$

Multiplying positive and negative numbers produces patterns:

Multiplying or dividing two numbers with the same sign yields a positive number.

Multiplying or dividing two numbers with different signs yields a negative number.

Any number multiplied by zero yields zero, and dividing by zero is not allowed.

Adding or subtracting zero has no effect on a number, and multiplying or dividing by one has no effect on a number.

When you add two numbers together, you form a *sum*.
When you subtract a number from another, you form a *difference*.
When you multiply two numbers, you form a *product*.
When you divide a number into another, you form a *quotient*.
When you “flip a number upside down”, you form its *reciprocal*.
Numbers that go evenly into another are its *divisors* or *factors*.
Numbers obtained by multiplying another with integers are its *multiples.*
Positive integers greater than one can be written as a product of *primes*.
GCF is Greatest Common Factor.
LCM is Least Common Multiple.

The *absolute value* of x, denoted by $|x|$, is equal to x if x is positive, and $-x$ if x is negative (so $|x|$ is never negative).

If n is a positive integer, then $n!$ denotes the product of all positive integers less than or equal to n. eg. 5!=5*4*3*2*1

The radical sign √ means "the nonnegative square root of".

You can't take the square root of a negative number.

The quadratic formula: If $ax^2+bx+c=0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sample questions

- Directions: In the following type of question, two quantities appear, one in Column A and one in Column B. You must compare them.
- Notes: Sometimes information about one or both of the quantities is centered above the two columns. If the same symbol appears in both columns, it represents the same thing each time.

Column A	Column B

m and *n* are positive integers

$mn = 25$

m	n

o The quantity in Column A is greater
o The quantity in Column B is greater
o The two quantities are equal
o The relationship cannot be determined from the information given

Strategy: Try some numbers... Make a couple of quick guesses.

- Think of two numbers that are multiplied to make 25.
- 5*5 = 25, so neither A nor B is greater
- 1*25 = 25, so they aren't necessarily equal

o The relationship cannot be determined from the information given

Column A		Column B
	$ab = 0$	
$(a + b)^2$		$(a - b)^2$

- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

Strategy choices

- Perform the algebraic operations indicated.
- A: $a^2 + 2ab + b^2$.
- B: $a^2 - 2ab + b^2$.
- If $ab = 0$, then these quantities are both $= a^2 + b^2$.

o The two quantities are equal

- Make a couple of quick guesses.
- $3*0 = 0$
- $(3 + 0)^2 = (3)^2 = 9$
- $(3 - 0)^2 = (3)^2 = 9$
- $0*2 = 0$
- $(0 + 2)^2 = (2)^2 = 4$
- $(0 - 2)^2 = (-2)^2 = 4$

o The two quantities are equal

The sum or difference of two odd or two even integers is even.
The sum or difference of an odd and an even integer is odd.
The product of two even or an even and an odd integer is even.
The product of two odd integers is odd.

$$b^n b^m = b^{n+m}$$

$$\frac{b^n}{b^m} = b^{n-m}$$

$$\left(b^n\right)^m = b^{nm}$$

$$b^n c^n = \left(bc\right)^n$$

$$b^0 = 1$$

$$b^1 = b$$

$$b^{-1} = \frac{1}{b}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$a(b+c)=ab+ac \quad a(b-c)=ab-ac$$

$$\frac{b+c}{a}=\frac{b}{a}+\frac{c}{a} \qquad \frac{b-c}{a}=\frac{b}{a}-\frac{c}{a}$$

$$a<b \Rightarrow \quad a+c<b+c$$

$$a<b \Rightarrow \quad a-c<b-c$$

$$a<b; c<d \Rightarrow a+c<b+d$$

Inequalities

- Be careful when multiplying or dividing an inequality: operations with negative numbers can change the direction of an inequality.
- Multiplying and dividing by positive numbers do not change the direction of an inequality.
- Addition and subtraction do not change the direction of an inequality.

Integers aren't the only numbers out there!

- Don't forget that there is more than one number between 4 and 6.
- (4.1, 5, 5.99999, $(2.1)^2$, ...)
- An easy trap to fall into is believing that if the question uses integers, the solutions must also use integers.
- "a number" could be any real number.

Column A	Column B

$$a + b = 24$$
$$a - b = 25$$

b	0

o The quantity in Column A is greater
o The quantity in Column B is greater
o The two quantities are equal
o The relationship cannot be determined from the information given

- How could the difference of two numbers be greater than the sum?
- Don’t forget that sometimes numbers are not positive integers.
- *b* must be a negative number if

  $a + b < a - b$.

o The quantity in Column B is greater

Fractions can be expressed as decimals by dividing the numerator by the denominator.

If the numerator is smaller, the decimal will begin with 0., but if the denominator is smaller, the decimal will be bigger than one.

Zeros can be added to the end of a decimal expansion to make comparisons easier: comparing 2.50000 and 2.51234 instead of comparing 2.5 and 2.51234

To add and subtract decimals, make sure you line up the decimal points.

To multiply decimals, pretend they are integers and perform the calculation, then count the total number of digits after the decimal point before the operation, and place the decimal point that many to the left in the result.

To divide decimals, move the decimal point right in both decimals until you have a pair of integers (add zeros if needed).

To multiply fractions, write them side-by-side and multiply across the top and bottom.

To divide fractions, invert the denominator fraction and multiply.

To add and subtract fractions, first find a common denominator. The common denominator is the least common multiple of the two denominators. If you feel rushed, multiply the denominators, add or subtract, then reduce the results.

- A quarter is worth 25 cents. Three quarters are 75 cents. ¾ = 0.75
- Fifteen minutes is a quarter of an hour. Three quarters of an hour is 45 minutes. ¾ = 45/60
- Any whole number can be written as a fraction by dividing by one. 37 = 37/1
- A fraction of a whole number is obtained by multiplication. Half an hour = ½ of 60 = ½ * 60/1 = 60/2 = 30/1 = 30

Column A	Column B
$\frac{\sqrt{24}}{2}$	$\frac{6}{\sqrt{6}}$

- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

- To compare two fractions, we look for a common denominator.

$$\frac{\sqrt{24}}{2} = \frac{\sqrt{24}\sqrt{6}}{2\sqrt{6}} = \frac{\sqrt{144}}{2\sqrt{6}}$$

$$\frac{6}{\sqrt{6}} = \frac{2*6}{2\sqrt{6}} = \frac{12}{2\sqrt{6}}$$

oThe two quantities are equal

To convert decimals to percents, multiply by 100 and add a % sign.

To convert percents to decimals, divide by 100 and remove the % sign.

a% of b = b% of a

Percent increase is (actual increase) divided by (original amount) * 100%

Percent decrease is (actual decrease) divided by (original amount) * 100%

To increase by *k*%, multiply by (1 + *k*%)

To decrease by *k*%, multiply by (1 - *k*%)

In 1980, the cost of p pounds of potatoes was d dollars. In 1990, the cost of $2p$ pounds of potatoes was $\frac{1}{2}d$ dollars. By what percent did the price of potatoes decrease from 1980 to 1990?

- 25%
- 50%
- 75%
- 100%
- 400%

- $2p$ costs ½ d
- p costs ¼ d
- Price has gone from d for p pounds to ¼ d for p pounds.
- The amount of decrease is

  ¾ = 0.75 = 75%

o 75%

Column A	Column B
65% of *a*	2/3 of *a*

o The quantity in Column A is greater
o The quantity in Column B is greater
o The two quantities are equal
o The relationship cannot be determined from the information given

- I’ll convert the fraction to a decimal, 2/3 = 0.6666...
- Compare to 65% = 0.65

o The quantity in Column B is greater

- Ratios are not the same as fractions. A ratio of 1 to 3 boys to girls means there is 1 boy for every 3 girls. Alternately, we can say this population has $\frac{1}{4}$ boys and $\frac{3}{4}$ girls.
- When considering rates, use the units as a guide; to find speed, think about miles per hour and get the fraction miles/hours or distance/time.

If *a* is increased by 25% and *b* is decreased by 25%, the resulting numbers will be equal. What is the ratio of *a* to *b*?

- 3/5
- 3/4
- 1/1
- 4/3
- 5/3

- $a + 25\%a = 1.25a$
- $b – 25\%b = 0.75b$
- $1.25a = 0.75b$
- a/b = 0.75/1.25
- $0.75 is 3 quarters
- $1.25 is 5 quarters
  - 3/5

The average (is the arithmetic mean) is calculated by adding up all the data points and dividing by the number of data points.

The median is the middle number in a list, or the average of the two middle numbers.

The mode is the number in a list that occurs most often.

Column A	Column B

$$a + 2b = 6d$$
$$c - b = 5d$$

The average (arithmetic mean) of a, b, c, and d	$3d$

- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

- $a + 2b = 6d$

  $c - b = 5d$ added together
- $a + b + c = 11d$
- The mean is $(a + b + c + d)/4$
- Substituting $(11d + d)/4 = (12d)/4 = 3d$

o The two quantities are equal

What is the average (arithmetic mean) of 3^{30}, 3^{60}, and 3^{90}?

- o 3^{60}
- o 3^{177}
- o $3^{10} + 3^{20} + 3^{30}$
- o $3^{27} + 3^{57} + 3^{87}$
- o $3^{29} + 3^{59} + 3^{89}$

- Sum the three and divide by three
- $(3^{30} + 3^{60} + 3^{90})/3$
- Distribute $(3^{30})/3 + (3^{60})/3 + (3^{90})/3$
- Exponent rules

$$3^{30}/3^{1} + 3^{60}/3^{1} + 3^{90}/3^{1}$$

o $3^{29} + 3^{59} + 3^{89}$

When adding and subtracting polynomials, make sure to keep track of parentheses and signs. Combine only like terms.

When multiplying polynomials, make sure to distribute fully. (Use FOIL)

When dividing polynomials by monomials, distribute and divide one term at a time.

Keep equations balanced.

If you have multiple equations, try adding them together.

Read questions carefully and only look for the answer you need, not all the possible information.

Jordan has taken 5 math tests so far this semester. If he gets a 70 on his next test, it will lower the average (arithmetic mean) of his test scores by 4 points. What is his average now?

- o 74
- o 85
- o 86
- o 90
- o 94

- His current average is (sum)/5
- 5 * current average = sum
- His future average will be (sum +70)/6
- Substituting, (5 * current + 70)/6 = future
- (5 * current + 70)/6 = current – 4
- (5 * current + 70) = 6 * (current – 4)
- 5current + 70 = 6current – 24

o 94

Column A | Column B

A school group charters three identical buses and occupies 4/5 of the seats. After ¼ of the passengers leave, the remaining passengers use only two of the buses.

Column A	Column B
The fraction of the seats on the two buses that are now occupied	9/10

- The quantity in Column A is greater
- The quantity in Column B is greater
- The two quantities are equal
- The relationship cannot be determined from the information given

- There are x seats on each bus. They started using 4/5 of $3x$ seats. $(4/5) * (3x) = (12/5)\,x$.
- When ¼ leave, ¾ remain. That is (3/4) of $(12/5)\,x$. $(3/4) * (12/5)\,x = (9/5)\,x$.
- There are still two buses, so on each bus there are $(1/2) * (9/5)\,x$

o The two quantities are equal

If $1/a + 1/a + 1/a = 12$, then $a =$

- o 1/12
- o 1/4
- o 1/3
- o 3
- o 4

- $1/a + 1/a + 1/a = 12$
- $3/a = 12$
- $3 = 12a$
- $3/12 = a$
  - 1/4

<u>Column A</u>	<u>Column B</u>

$$x^5 = (13/17)$$

x	$(13/17)^5$

- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

- $x^5 = (13/17)$
- We are comparing x and $(13/17)^5$
- Substituting, we are comparing x and $(x^5)^5$
- $0 < x < 1$ because $0 < (13/17) < 1$
- So, $x^{25} < x$

o The quantity in Column A is greater

Acute angles measure less than 90º

A right angle measures 90º

Obtuse angles measure between 90º and 180º

Straight angles measure 180º

The sum of angle measures lying along a straight line is 180º

The sum of angle measures around a circle is 360º

If two lines are perpendicular, they meet at 90º angles.

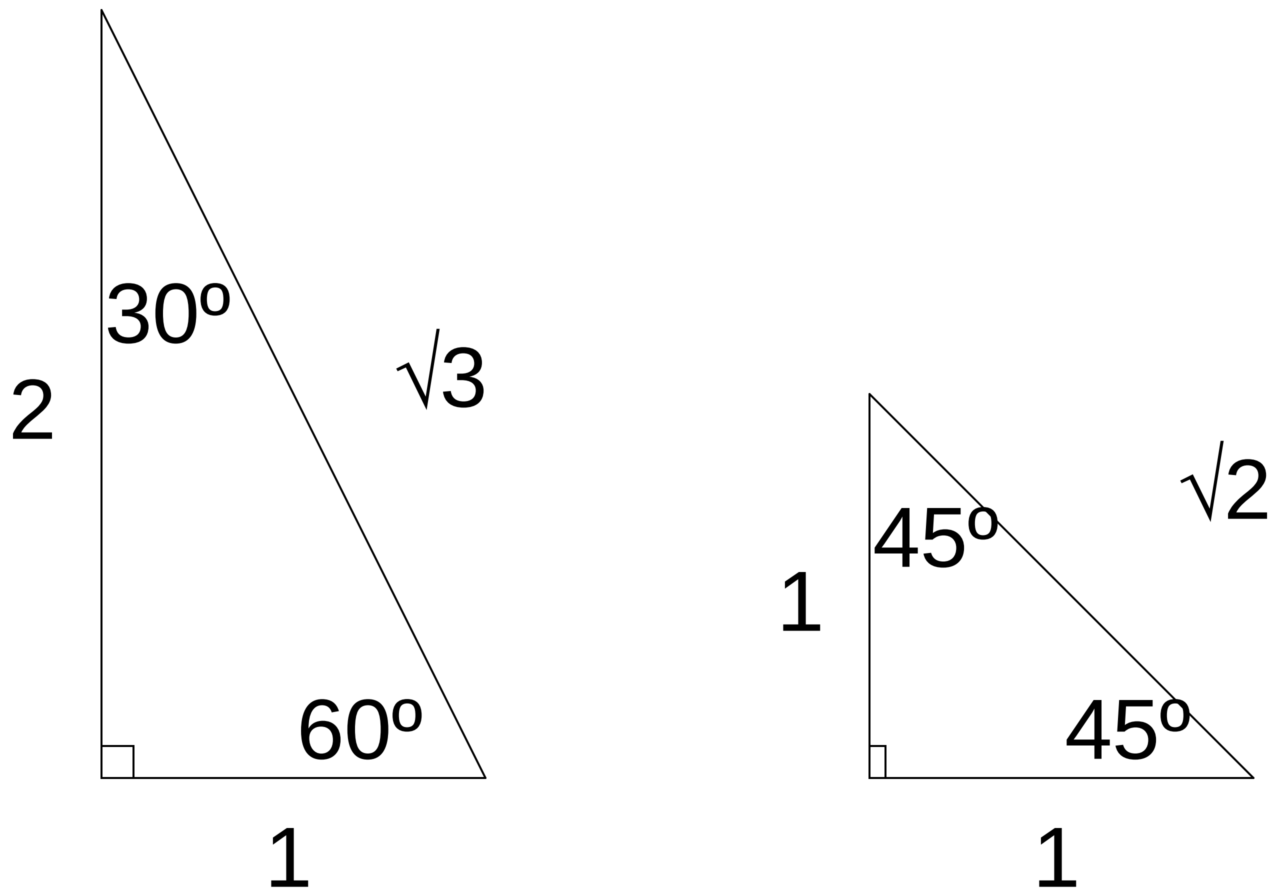
Vertical angles have the same measure.

The sum of measures in a triangle is 180º

The longest side of any triangle is opposite the largest angle, the smallest side is opposite the smallest angle.

If two sides of a triangle have the same length, then their opposite angles have the same measure.

$a^2 + b^2 = c^2$ where *c* is the hypotenuse (opposite the right angle) and *a* and *b* are the two legs of the triangle.


30º
2
√3
60º
1
45º
1
√2
45º
1

The area of a triangle is ½ $b*h$

The perimeter is the length of the three sides, summed.

The perimeter of a quadrilateral is the sum of the four sides.

The area of a parallelogram is $b*h$

The area of a square is s^2 or ½ d^2

The area of a rectangle is $l*w$

The area of trapezoid is ½ $(b_1 + b_2)*h$

For a circle, *d* is diameter and *r* is radius,

$$d = 2r$$

Circumference is $2\pi r$ or πd

Area of a circle is πr^2

Volume of a rectangular solid is l*w*h

Surface area is 2(l*w + l*h + w*h)

If d is a diagonal of a box, $d^2 = l^2 + w^2 + h^2$

Volume of a cylinder is $\pi r^2 h$

The distance between two points is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope is rise / run

= (y difference) / (x difference)

Probability is also a rational function

= (number of ways “it” happens) / (number of possible things that happen)

A probability of zero means it will NOT happen

A probability of one means it WILL happen

All other probabilities are between zero and one.

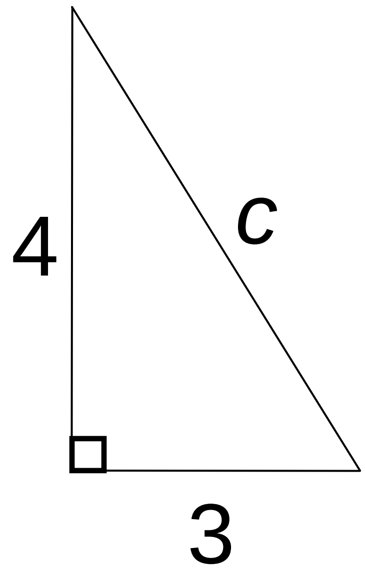
The Center City Little League is divided into *d* divisions. Each division has *t* teams, and each team has *p* players. How many players are there in the entire league?

- ○ $d + t + p$
- ○ dtp
- ○ $\dfrac{pt}{d}$
- ○ $\dfrac{dt}{p}$
- ○ $\dfrac{d}{pt}$

- Each team has *p* players, and there are *t* teams in a division.
- Each division has *p*t* players.
- There are *d* divisions.
- The League has *p*t*d* players

  - dtp

Column A	Column B
c	5

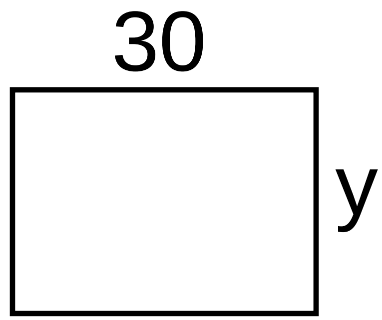


- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

- $3^2 + 4^2 = 9 + 16 = 25$
- $c^2 = 25$
- $5^2 = 25$

o The two quantities are equal

Column A	Column B
Area	500



- o The quantity in Column A is greater
- o The quantity in Column B is greater
- o The two quantities are equal
- o The relationship cannot be determined from the information given

- Figures are not always drawn to scale!
- *y* could be 20, but couldn’t it also be 25?

o The relationship cannot be determined from the information given

A bag contains 3 red, 4 white, and 5 blue marbles. Jason begins removing marbles from the bag at random, one at a time. What is the least number of marbles he must remove to be sure that he has at least one of each color?

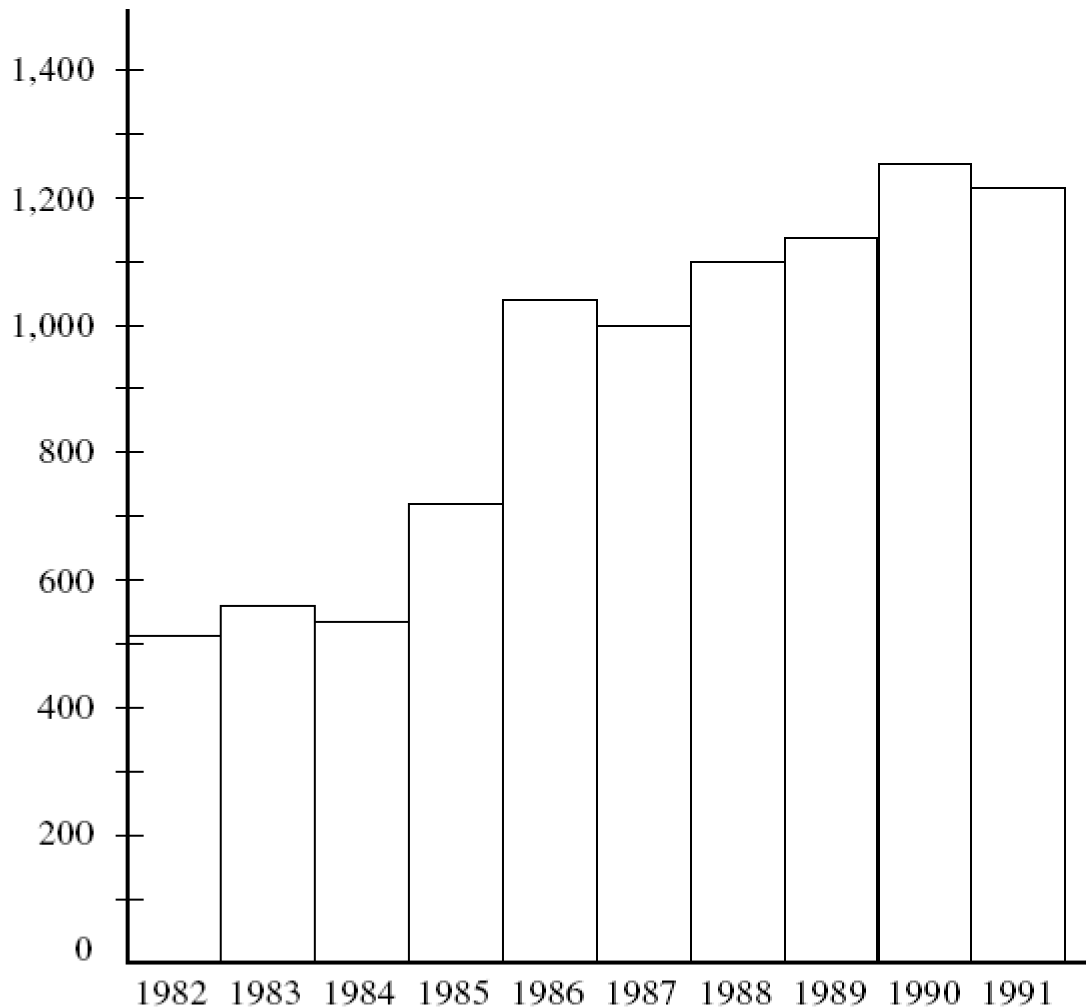
- 3
- 6
- 8
- 10
- 12

- This is a counting question, so we’ll count.
- The question asks for the least number, but it also asks to make sure he gets one of each color.
- Look at the worst-case scenario:

  If the first 5 marbles are all blue, he’ll have to keep pulling, if the next 4 are all white, he’ll have to pull one more. 5 + 4 + 1

o 10

NUMBER OF GRADUATE STUDENT APPLICANTS AT UNIVERSITY *X*, 1982-1991



In which of the following years did the number of graduate student applicants increase the most from that of the previous year?

(A) 1985

(B) 1986

(C) 1988

(D) 1990

(E) 1991

Where in the graph is the biggest jump?

- 1986

Which of the following statements can be inferred from the graph?

I. The number of graduate student applicants more than doubled from 1982 to 1991.

II. For each of the years 1983 to 1991, inclusive, the number of graduate student applicants was greater than that of the previous year.

III. The greatest number of graduate students attended University *X* in 1990.

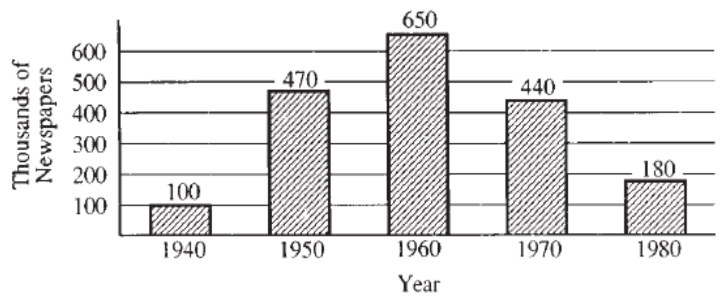
(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

Consider statements one at a time
(I) Number of students more than doubled: Less than 600 to start, around 1200 at the end – YES
(II) The number of students did NOT increase every year – NO
(III) Read the label! Chart is about the number of applicants, not the number attending – NO

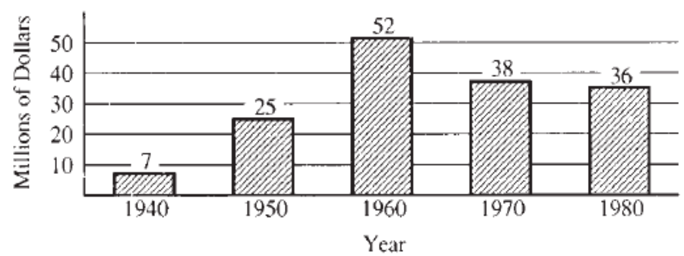
Conclusion: I only

Questions 21-25 refer to the following graphs.

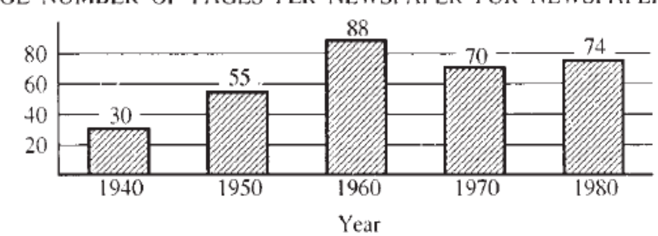
AVERAGE DAILY CIRCULATION FOR NEWSPAPER X



TOTAL YEARLY ADVERTISING REVENUE FOR NEWSPAPER X



AVERAGE NUMBER OF PAGES PER NEWSPAPER FOR NEWSPAPER X



21. In how many of the years shown was the average number of pages per newspaper at least twice as much as the average in 1940 ?

(A) Four
(B) Three
(C) Two
(D) One
(E) None

We need to know:

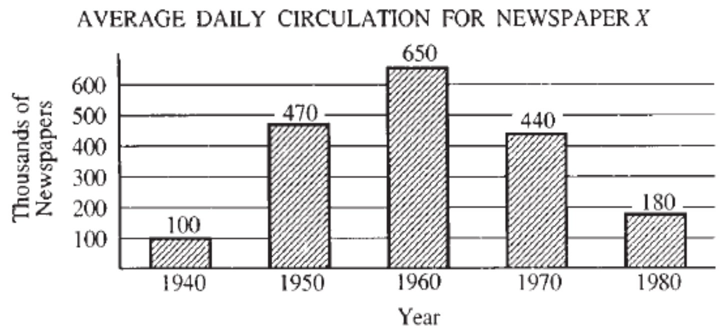
-Which chart to look at (average PAGES)

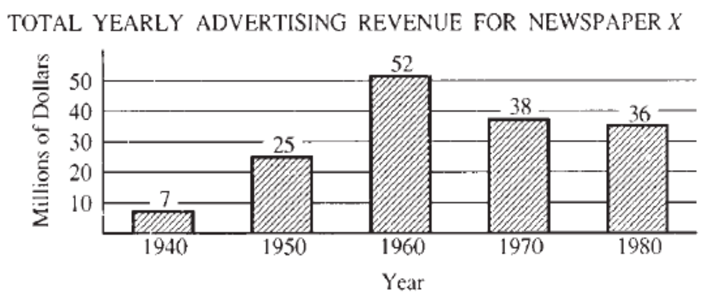
-Number of pages in 1940 (30)

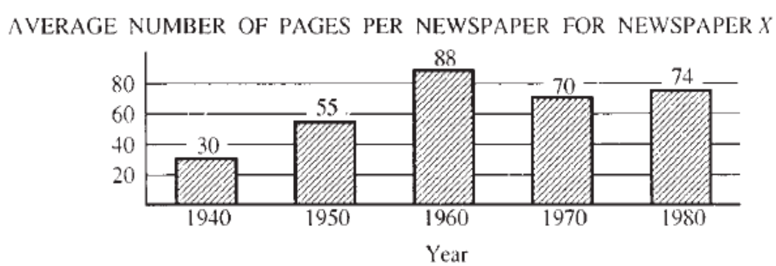
-Count how many years have at LEAST 60 pages (1960, 1970, 1980)

Conclusion: (B) Three

Questions 21-25 refer to the following graphs.







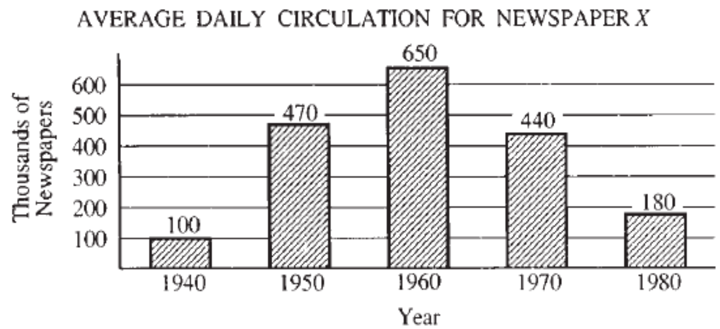
22. In 1950, if the printing cost per newspaper was $0.05, what would have been the total cost of printing the average daily circulation?

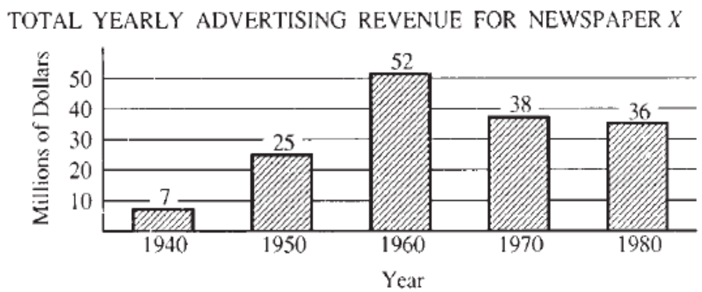
(A) $32,500
(B) $26,000
(C) $23,500
(D) $22,000
(E) $2,600

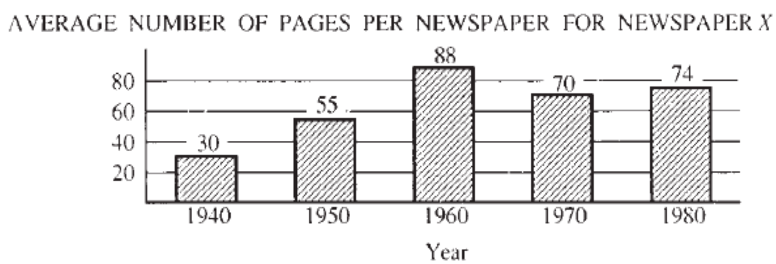
We need to know:
-Which chart to look at (average DAILY CIRCULATION)
-Average circulation in 1950 (470,000)
-.05 x 470,000 = 23500.00

Conclusion: (C) $23,500

Questions 21-25 refer to the following graphs.







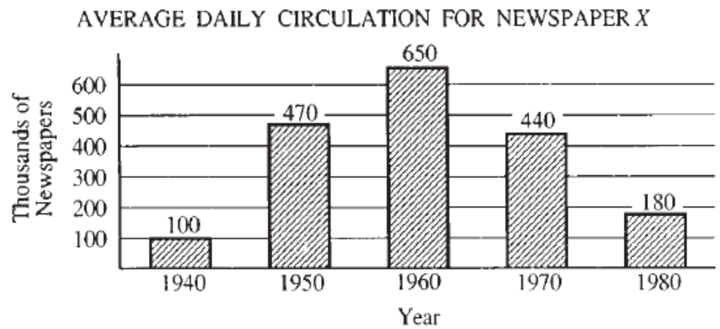
23. In 1980 the number of dollars of advertising revenue was how many times as great as the average daily circulation?

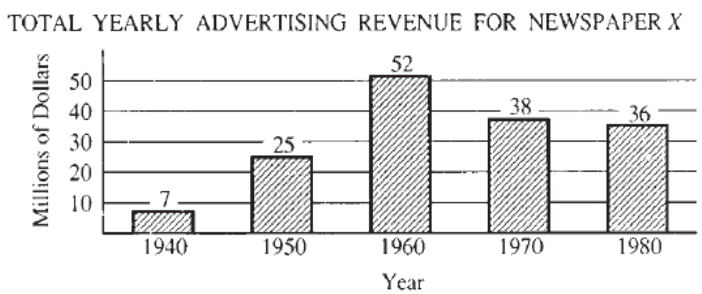
(A) 500
(B) 200
(C) 100
(D) 50
(E) 20

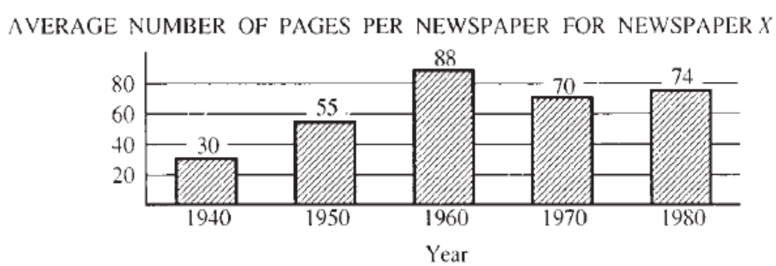
We need to know:
-Advertising revenue in 1980 (36 mill.)
-Average circulation in 1980 (180,000)
-36,000,000 is how many times as great as 180,000?
36,000,000 / 180,000 = 200

Conclusion: (B) 200

Questions 21-25 refer to the following graphs.







24. The percent decrease in average daily circulation from 1960 to 1970 was approximately

(A) 10%
(B) 12%
(C) 20%
(D) 26%
(E) 32%

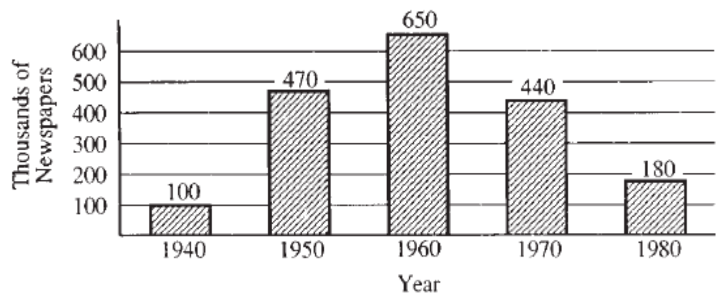
We need to know:
-Average circulation in 1960 (650,000)
-Average circulation in 1970 (440,000)
-Percent decrease is (actual decrease) divided by (original amount) * 100%

So: 210,000 / 650,000 * 100%
Estimate: 21/65 ~ 1/3 (33%)

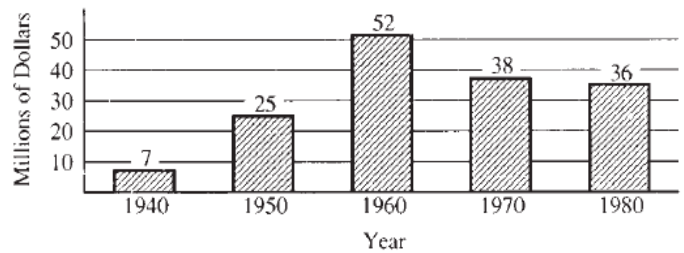
Conclusion: (E) 32%

Questions 21-25 refer to the following graphs.

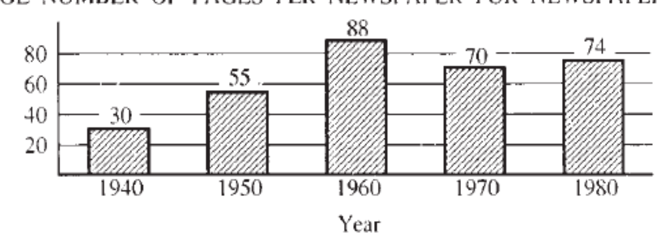
AVERAGE DAILY CIRCULATION FOR NEWSPAPER *X*



TOTAL YEARLY ADVERTISING REVENUE FOR NEWSPAPER *X*



AVERAGE NUMBER OF PAGES PER NEWSPAPER FOR NEWSPAPER *X*



25. Which of the following statements can be inferred from the data?

I. The greatest increase in total yearly advertising revenue over any 10-year period shown was \$27 million.

II. In each of the 10-year periods shown in which yearly advertising revenue decreased, average daily circulation also decreased.

III. From 1970 to 1980 the average number of pages per newspaper increased by 10.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

Consider them one at a time:
-(I) Greatest increase in yearly advertising revenue over 10-yr period: 27 million from 1950-1960 - YES
-(II) Revenue decreased from 1960 to 1970 and 1970 to 1980, and avg daily circulation decreased, too - YES
-(III) Based on choices, can’t be true, and it isn’t (increase was 4 pages)

Conclusion: (D) I and II

THE END!

True Facts:

•All of the numbers used in the GRE are real numbers – this means there are no complex or imaginary numbers.
•Numbers less than zero are called negative, and those greater than zero are called positive.
•Negative numbers are always less than positive numbers. eg. $-3 < 3$
•Multiplying positive and negative numbers produces patterns:
•Multiplying or dividing two numbers with the same sign yields a positive number.
•Multiplying or dividing two numbers with different signs yields a negative number.
•Any number multiplied by zero yields zero, and dividing by zero is not allowed.
•Adding or subtracting zero has no effect on a number, and multiplying or dividing by one has no effect on a number.
•When you add two numbers together, you form a *sum*. When you subtract a number from another, you form a *difference*.
•When you multiply two numbers, you form a *product*.
•When you divide a number into another, you form a *quotient*.
•When you "flip a number upside down", you form its *reciprocal*.
•Numbers that go evenly into another are its *divisors* or *factors*.
•Numbers obtained by multiplying another with integers are its *multiples.*
•Positive integers greater than one can be written as a product of *primes*.
•GCF is Greatest Common Factor.
•LCM is Least Common Multiple.
•The sum or difference of two odd or two even integers is even.
•The sum or difference of an odd and an even integer is odd.
•The product of two even or an even and an odd integer is even.
•The product of two odd integers is odd.
•Be careful when multiplying or dividing an inequality: operations with negative numbers can change the direction of an inequality.
•Multiplying and dividing by positive numbers do not change the direction of an inequality.
•Addition and subtraction do not change the direction of an inequality.
•To multiply fractions, write them side-by-side and multiply across the top and bottom.
•To divide fractions, invert the denominator fraction and multiply.
•To add and subtract fractions, first find a common denominator. The common denominator is the least common multiple of the two denominators. If you feel rushed, multiply the denominators, add or subtract, then reduce the results.
•A quarter is worth 25 cents. Three quarters are 75 cents. $3/4 = 0.75$
•Fifteen minutes is a quarter of an hour. Three quarters of an hour is 45 minutes. $3/4 = 45/60$

•Any whole number can be written as a fraction by dividing by one. 37 = 37/1
•A fraction of a whole number is obtained by multiplication. Half an hour = ½ of 60 = ½ * 60/1 = 60/2 = 30/1 = 30
•Ratios are not the same as fractions. A ratio of 1 to 3 boys to girls means there is 1 boy for every 3 girls. Alternately, we can say this population has ¼ boys and ¾ girls.
•When considering rates, use the units as a guide; to find speed, think about miles per hour and get the fraction miles/hours or distance/time.
•Acute angles measure less than 90º
•A right angle measures 90º
•Obtuse angles measure between 90º and 180º
•Straight angles measure 180º
•The sum of angle measures lying along a straight line is 180º
•The sum of angle measures around a circle is 360º
•If two lines are perpendicular, they meet at 90º angles.
•The average (is the arithmetic mean) is calculated by adding up all the data points and dividing by the number of data points.
•The median is the middle number in a list, or the average of the two middle numbers.
•The mode is the number in a list that occurs most often.
•Slope is rise / run = (y difference) / (x difference)
•Probability is also a rational function = (number of ways “it” happens) / (number of possible things that happen)
•A probability of zero means it will NOT happen
•A probability of one means it WILL happen
•All other probabilities are between zero and one.
•The absolute value of x, written |x|, is equal to x if x is positive and –x if x is negative (so |x| is never negative). |-3|=3
•If n is a positive integer, then n! denotes the product of all positive integers less than or equal to n. 5! = 5*4*3*2*1=120
•The radical sign √ means the nonnegative square root of a number.
•You can’t take the square root of a negative number.
•The quadratic formula: If $ax^2+bx+c=0$, then: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$b^n b^m = b^{n+m}$$

$$a(b+c) = ab + ac$$

$$a(b-c) = ab - ac$$

$$\frac{b^n}{b^m} = b^{n-m}$$

$$\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$$

$$\frac{b-c}{a} = \frac{b}{a} - \frac{c}{a}$$

$$\left(b^n\right)^m = b^{nm}$$

$$a < b \Rightarrow \quad a + c < b + c$$

$$b^n c^n = \left(bc\right)^n$$

$$a < b \Rightarrow \quad a - c < b - c$$

$$b^0 = 1$$

$$a < b; c < d \Rightarrow a + c < b + d$$

$$b^1 = b$$

$$b^{-1} = \frac{1}{b}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

- $a^2 + b^2 = c^2$ where *c* is the hypotenuse (opposite the right angle) and *a* and *b* are the two legs of the triangle.
- The area of a triangle is ½ b*h
- The perimeter of a triangle is the length of the three sides, summed.
- The perimeter of a quadrilateral is the sum of the four sides.
- The area of a parallelogram is b*h
- The area of a square is s^2 or ½ d^2
- The area of a rectangle is l*w
- The area of trapezoid is ½ $(b_1 + b_2)$*h

- For a circle, *d* is diameter and *r* is radius,

$$d = 2r$$

- Circumference is $2\pi r$ or πd
- Area of a circle is πr^2
- Volume of a rectangular solid is l*w*h
- Surface area is 2(l*w + l*h + w*h)
- If d is a diagonal of a box, $d^2 = l^2 + w^2 + h^2$
- Volume of a cylinder is $\pi r^2 h$
- The distance between two points is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Data missing for questions in the slides that follow

A family’s unmet need (which must be covered by a financial aid package) is defined to be the total cost of attending an institution of higher education minus the expected family contribution. What is the unmet need of a family whose income is $55,000 and who has a child attending a 4-year public university?

- $700
- $3300
- $6800
- $7500
- $12,500

- Family income of $55,000 makes the expected contribution near $7,500.
- The cost of attending a 4-year public university is $10,800.
- The difference is the unmet need: $10,800 - $7,500 = $3,300

o $3300

If family A has an income of $95,000 per year, and family B has an income of $35,000 per year, and each has a child attending a 4-year public university, to the nearest $1000, how much more would family A be expected to pay than family B?

- $4000
- $7000
- $10,000
- $12,000
- $15,000

- The $95,000 family is expected to contribute about $16,000.
- The $35,000 family is expected to contribute about $3,800.
- The $95,000 family will have to pay $10,800, and the $35,000 will contribute $3,800.
- $10,800 - $3,800

o $7000

To the nearest million, how many more students were enrolled in school – both public and private, preK-12 – in 1970 than in 1988?

- 3,000,000
- 6,000,000
- 10,000,000
- 44,000,000
- 51,000,000

- From the top graph, we get:

	1970	1988
Public PreK-8	33,000,000	28,000,000
Public 9-12	13,000,000	12,000,000
Private PreK-8	4,000,000	4,000,000
Private 9-12	1,000,000	1,000,000
Total	51,000,000	45,000,000

o 6,000,000

In 1988 there were 40,000,000 public school students in the United States, of whom 22% lived in the West. Approximately, how many public school students are projected to be living in the West in 2008?

- 9,000,000
- 12,000,000
- 15,000,000
- 24,000,000
- 66,000,000

- 22% of 40,000,000 = 0.22 * 40,000,000 = 8,800,000 were in the West
- This grows by 27% in the next decade, so that’s a bit more than 25% (an easier number).
- 8,800,000 + 2,200,000 = 11,000,000
- This grows again by 10%
- 11,000,000 + 1,100,000 = 12,100,000

o 12,000,000

- MISSING PROBLEM THAT GOES WITH EXPLANATION ON NEXT SLIDE

- Both squares have an area of $12^2 = 144$. We need to figure out which circles have greater total area.
- In figure 1, 4 circles, each having diameter half of 12 = 6, radius is half of 6 = 3. Circles have area $\pi r^2 = \pi(3)^2 = 9\pi$. There are 4 circles, $4*9\pi = 36\pi$
- In figure 2, 9 circles, each having diameter a third of 12 = 4, radius is half of 4 = 2. Area is $\pi(2)^2 = 4\pi$. There are 9 circles, $9*4\pi = 36\pi$

o The two quantities are equal

Review of the Quantitative Section

Overview

The quantitative section measures your basic mathematical skills, your understanding of elementary mathematical concepts, and your ability to reason quantitatively and solve problems in a quantitative setting. There is a balance of questions requiring arithmetic, algebra, geometry, and data analysis. These are content areas usually studied in high school.

Arithmetic

Questions may involve arithmetic operations, powers, operations on radical expressions, estimation, percent, absolute value, properties of integers (e.g., divisibility, factoring, prime numbers, odd and even integers), and the number line.

Algebra

Questions may involve rules of exponents, factoring and simplifying algebraic expressions, understanding concepts of relations and functions, equations and inequalities, solving linear and quadratic equations and inequalities, solving simultaneous equations, setting up equations to solve word problems, coordinate geometry, including slope, intercepts, and graphs of equations and inequalities, and applying basic algebra skills to solve problems.

Geometry

Questions may involve parallel lines, circles, triangles (including isosceles, equilateral, and $30°–60°–90°$ triangles), rectangles, other polygons, area, perimeter, volume, the Pythagorean Theorem, and angle measure in degrees. The ability to construct proofs is not measured.

Data Analysis

Questions may involve elementary probability, basic descriptive statistics (mean, median, mode, range, standard deviation, percentiles), and interpretation of data in graphs and tables (line graphs, bar graphs, circle graphs, frequency distributions).

Math Symbols and Other Information

The following information applies to all questions in the quantitative sections.

- These common math symbols may be used:
  - $x < y$ (x is less than y)
  - $x \neq y$ (x is not equal to y)
  - $\sqrt{x}$ (the nonnegative square root of x, where $x \geq 0$)
  - $|x|$ (the absolute value of x, where x is a real number)
  - $n!$ (n factorial: the product of the first n positive integers)
  - $m \parallel n$ (line m is parallel to line n)
  - $m \perp n$ (line m is perpendicular to line n)
  - 

    ($\angle ABC$ is a right angle)
- Numbers: all numbers used are real numbers.
- Figures:
  - the positions of points, angles, regions, etc., can be assumed to be in the order shown; angle measures are positive
  - a line shown as straight can be assumed to be straight
  - figures lie in a plane unless otherwise indicated
  - do not assume figures are drawn to scale unless stated

It is important to familiarize yourself with the basic mathematical concepts in the GRE General Test. The publication *Math Review* is available for free download on the GRE Web site at www.gre.org/pracmats.html and provides detailed information on the content of the quantitative section.

The quantitative section contains the following question types:

- Quantitative Comparison Questions
- Problem Solving – Discrete Quantitative Questions
- Problem Solving – Data Interpretation Questions

Questions emphasize understanding basic principles and reasoning within the context of given information.

How the Quantitative Section is Scored

The quantitative section of the paper-based General Test is scored the same way as the verbal section. First, a raw score is computed. The raw score is the number of questions for which the best answer choice was given. The raw score is then converted to a scaled score through a process known as equating. The equating process accounts for differences in difficulty among the different test editions; thus a given scaled score reflects approximately the same level of ability regardless of the edition of the test that was taken.

Quantitative Comparison Questions

Quantitative comparison questions measure your ability to:

- reason quickly and accurately about the relative sizes of two quantities
- perceive that not enough information is provided to make such a decision

Directions*

Each of the sample questions consists of two quantities, one in Column A and one in Column B. There may be additional information, centered above the two columns, that concerns one or both of the quantities. A symbol that appears in both columns represents the same thing in Column A as it does in Column B.

You are to compare the quantity in Column A with the quantity in Column B and decide whether:

(A) The quantity in Column A is greater.
(B) The quantity in Column B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

Note: Since there are only four choices, NEVER MARK (E).**

Sample Questions

	Column A	Column B
1.	9.8	$\sqrt{100}$
2.	$(-6)^4$	$(-6)^5$

Strategies for Answering

- Avoid extensive computation if possible. Try to estimate the answer.
- Consider all kinds of numbers before deciding. If under some conditions Column A is greater than Column B and for others, Column B is greater than Column A, choose "the relationship cannot be determined from the information given," and go to the next question.
- Geometric figures may not be drawn to scale. Comparisons should be made based on the given information, together with your knowledge of mathematics, rather than on exact appearance.

Answer to Question 1

$\sqrt{100}$ denotes 10, the positive square root of 100. (For any positive number x, $\sqrt{x}$ denotes the positive number whose square is x.) Since 10 is greater than 9.8, the best answer is (B). It is important not to confuse this question with a comparison of 9.8 and x where $x^2 = 100$. The latter comparison would yield (D) as the correct answer because $x^2 = 100$ implies that either $x = 10$ or $x = -10$, and there would be no way to determine which value x would actually have.

Answer to Question 2

Since $(-6)^4$ is the product of four negative factors, and the product of an even number of negative numbers is positive, $(-6)^4$ is positive. Since the product of an odd number of negative numbers is negative, $(-6)^5$ is negative. Therefore, $(-6)^4$ is greater than $(-6)^5$ since any positive number is greater than any negative number. The best answer is (A). It is not necessary to calculate that $(-6)^4 = 1{,}296$ and that $(-6)^5 = -7{,}776$ in order to make the comparison.

Problem Solving – Discrete Quantitative Questions

Discrete quantitative questions measure

- basic mathematical knowledge
- your ability to read, understand, and solve a problem that involves either an actual or an abstract situation

Directions*

Each of the following questions has five answer choices. For each of these questions, select the best of the answer choices given.

* The directions are presented as they appear on the actual test.
** The answer sheet contains five choices for the verbal and quantitative sections.

Sample Question

When walking, a certain person takes 16 complete steps in 10 seconds. At this rate, how many complete steps does the person take in 72 seconds?

(A) 45
(B) 78
(C) 86
(D) 90
(E) 115

Strategies for Answering

- Determine what is given and what is being asked.
- Scan all answer choices before answering a question.
- When approximation is required, scan answer choices to determine the degree of approximation.
- Avoid long computations. Use reasoning instead, when possible.

Answer

72 seconds represents 7 ten-second intervals plus 2/10 of such an interval. Therefore, the person who takes 16 steps in 10 seconds will take (7.2) (16) steps in 72 seconds.

$$\begin{aligned}(7.2)(16) &= (7)(16) + (0.2)(16)\\ &= 112 + 3.2\\ &= 115.2\end{aligned}$$

Since the question asks for the number of complete steps, the best answer choice is (E).

Problem Solving – Data Interpretation Questions

Data interpretation questions measure your ability

- to synthesize information and select appropriate data for answering a question
- to determine that sufficient information for answering a question is not provided

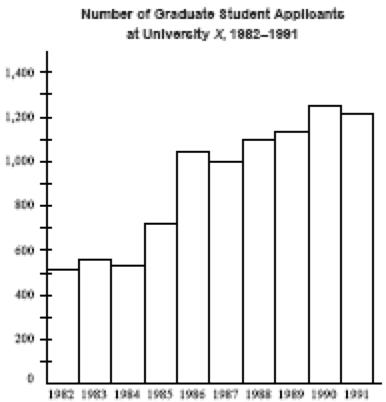
The data interpretation questions usually appear in sets and are based on data presented in tables, graphs, or other diagrams.

Directions*

Each of the following questions has five answer choices. For each of these questions, select the best of the answer choices given.

* The directions are presented as they appear on the actual test.

Sample Question



In which of the following years did the number of graduate student applicants increase the most from that of the previous year?

(A) 1985
(B) 1986
(C) 1988
(D) 1990
(E) 1991

Strategies for Answering

- Scan the set of data to see what it is about.
- Try to make visual comparisons and estimate products and quotients rather than perform computations.
- Answer questions only on the basis of data given.

Answer

This question can be answered directly by visually comparing the heights of the bars in the graph. The greatest increase in height between two adjacent bars occurs for the years 1985 and 1986. The best answer is (B).